A mathematical theory of accounting and discounted cash flows

Abstract

This paper develops a novel theory of accounting, in the form of a quantitative language, capable of reformulating, generalizing, and studying the discounted cash flow model, in plain accounting terms - going through the reformulation of the underlying future cash flows and their financing. This language reveals new facets of value creation mechanisms, specifically certain dynamic-financial limits (e.g., internal rate of return, cash flow sustainable growth rate) whose level is intrinsically tied to the shape of the fundamental accounting entries, describing the economic activities conducted by companies. Our findings, as to value creation mechanisms, correspond to various empirical observations on stock returns and its major factors (e.g., cash flows, payment terms, growth, inflation and financing rates). This correspondence suggests an original and complementary explanation to those advanced in the literature and finally suggests levers for managers to push back limits, increase cash flows, create more value and generate stock returns.

Keywords: Accounting; Discounted Cash flows; Value; Stock return.

JEL classifications: D46; G11; G12; M41

1. Introduction

The discounted cash flow (DCF) model is ubiquitous in finance and stands as a standard in the field of asset valuation (Damodaran 2007), as for its strong theoretical foundation and its wide range of applications (e.g. stocks, bonds). In recent years, this model went central in international accounting standards (e.g. IFRS, US GAAP) to value assets, when market prices are unavailable. This change marks the advent of a new and controversial "actuarial/forward-looking" phase in accounting thought (e.g., Cardao-Pito and Ferreira 2018; Markarian 2018). The influence of finance on accounting is clearly visible – this relationship can be deepened and in return accounting could well contribute to finance, given the nature of the information shared by these disciplines. Indeed, accounting gives an accurate measure of the *real* cash flows (CF), whose forecasts are at the heart of the DCF model. The potential mastery of *future* CF by accounting supposes a means of articulating its notions (e.g., entries, accounting balances) in analytical terms. This question is little addressed in the literature, and yet constitutes the starting point to formulate and study the general behavior of CF (real or future), to reformulate the DCF model and study its new facets. One could still assume that the foundations of DCF have been sufficiently studied, but this assumption reserves surprises, as noted by Damodaran (2007, p. 694): "*[...] the research into valuation models and metrics in finance is surprisingly spotty, with some aspects of valuation, such as risk assessment, being deeply analyzed and others, such as how best to estimate cash flows [...] not receiving the attention that they deserve.*" Indeed, CF are generally reduced to contingent symbols in models or assumed to be close to earnings. Givoly et al. (2009) and Pae and Yoon (2012) confirm this conception, qualifying financial analysts' CF forecasts as "naive" and "trivial" extensions of earnings forecasts and especially show that CF forecasts turn out to be of much poorer quality than earnings forecasts. This significant difference in behavior between earnings, CF and their models measures a crucial gap between the current measure of DCF and what it would be assuming real control of the underlying CF, real or future. This gap could mislead economic decisions and this paper proposes to reduce this gap by developing a mathematical theory of accounting, in the form of a quantitative language, capable of reformulating, generalizing, and studying the discounted cash flow model, in plain accounting terms - going through the reformulation of the underlying future CF and their financing. This language reveals new facets of value creation mechanisms, specifically certain dynamic-financial limits (e.g., internal rate of return, cash flow sustainable growth rate) whose level is intrinsically tied to the shape of the fundamental accounting entries, describing the economic activities conducted by companies. Our findings, as to value creation mechanisms, correspond to various empirical observations on stock returns and its major factors (e.g., cash flows, payment terms, growth, inflation and financing rates). This correspondence suggests an original and complementary explanation to those advanced in the literature and finally suggests levers for managers to push back limits, increase CF, create more value and generate stock returns.

We now detail the main stages of this project. First, we provide a precise and general formulation of CF, using a mathematical theory of accounting that can formulate and describe any economic activity. Indeed, the accounting *language* is capable of giving a faithful description of real economic activities, whatever their nature (e.g., industry, services). This description takes the form of balance sheets, allowing to describe naturally the evolution of real CF. The vast possibilities of accounting justify in our opinion to model it directly in the form of a fundamental quantitative language, so as to describe future or fictitious activities as well as real ones. From this perspective, we contribute to the stream of analytical research in accounting, led by Chambers, Mattessich, and Ijiri, notably (e.g., Gaffikin 2003, 2005), which aims to endow this language with axioms and matrix calculations that can model the evolution of balance sheets over time. The study background section presents and discusses recent models of balance sheets evolution (Arya et al. 1996; Girardi et al. 2011; Gentili and Giacomello 2017) that most closely relate to our study. Our theory differs from these works as it synthesizes the fundamental operations of accounting, that is, the successive basic accounting entries, to a compact form named *schemes*. Schemes can provide a complete accounting description of the basic and contingent *economic operations* (e.g., a single sale, purchase, a loan). We simulate the production of streams of accounting information, equivalent to a balance sheet evolving over time, by combining these schemes with the evolution of the number of underlying economic operations conducted over time. The mathematical operations involved, that is, the *discrete convolution products* (e.g., Gray 2006), provides a fundamental and general formulation of accounting balances, from which the calculation of CF naturally proceeds. The successive adjunction of the effects of financing and inflation to our accounting model of CF leads us not only to reformulate the DCF model, but to generalize and deepen it. Indeed, we contribute to the literature by providing the first formulation of DCF, described entirely in terms of fundamental accounting schemes - whose shape is specific to each economic activity and allows for the combination of prices and the various delays (specifically, payment terms, inventory, and production cycle duration). These new components and variables, described at the most granular level of economic operations and related schemes, offer the opportunity to study their outcomes in terms of CF and value creation, under the effects of the evolution of economic activity, its financing and the level of inflation. This step consists in a *reduction* and a *unification* (e.g., Van Riel and Van Gulick 2019) of accounting and finance domains, and allows to consider the latter from a more fundamental perspective. Subsequently, we show that the potential of value creation of economic activities is characterized by three dynamic-financial limits, all tied to the shape of the fundamental accounting scheme of economic activities and beyond which CF and DCF pass into negative territory. The first limit is the well-known "internal rate of return" (IRR), which corresponds to the rate of financing cancelling the value of DCF (and NPV). The second limit is currently known as a growth limit that can cancel CF (Hamman 1996; Churchill and Mullins 2001; Dreyer et al. 2013), as well as DCF, as shown in this manuscript. Then, we identify a third limit in the inflation domain, which may not have been identified in the literature, to which we contribute by proving that these three dynamic-financial limits are in fact one. In doing so, our theory offers the means to finely study the inner mechanisms of CF, DCF and their variables (e.g., schemes, growth, financing, and inflation).

We show how our accounting formulation of DCF, and the analysis of its behavior, provides explanations for various stylized facts about the drivers of stock returns. Studies have shown the reliability of the DCF model in estimating the market values of companies (e.g., Kaplan and Ruback 1995, 1996). Therefore, increases in the estimates provided by the DCF model should lead to an increase in the market value of companies and thus generate stock returns. With CF at the heart of DCF, the literature confirms the strong link between changes in CF, whether future (e.g., Vuolteenaho 2002; Fama and French 2008 ; Larrain and Yogo 2008 ; Chen et al. 2013 ; Jansen 2021) or actual (e.g., Da 2009 ; Hou et al. 2011 ; Novy-Marx 2013 ;

Ball et al. 2016 ; Foerster et al. 2017), on stock returns. Our results contribute in particular to the studies conducted on the significant effects of payment terms (customers and suppliers) on stock returns (e.g., Zeidan and Shapir 2017 ; Wang 2019 ; Lin and Lin 2021). Indeed, our notion of scheme captures these different payment terms and makes it possible to quantify their impact on the dynamic-financial limits, affecting the potential of CF, DCF and finally stock returns. Finally, our analysis of the effects of these dynamic-financial limits provides an original explanation for the erosive effects of growth, inflation and financing rates on stock returns - effects that have been documented in the literature and for which different explanations have been advanced. The remainder of this paper is organized as follows. Section 2 presents the standard method in finance for modeling CF, as well as the attempts made by accounting research to give it an analytical formulation. Section 3 presents our mathematical theory of accounting. In Section 4, we successively reformulate CF and DCF. In Section 5, we study the dynamic behavior and limits of CF and DCF. Section 6 bridges our theory with market finance, regarding value creation and stock returns explained in terms of factors. In Section 7, we conclude the study.

2. Study background

In foreword, we specify that the CF used in the DCF model are the same *Free Cash Flows*, equal to the operating cash flows generated by the company's activity, diminished by the amount of investments. The difficulty lies in operations CF, involving complex factors developing over the time, such as prices, payment terms and the dynamics of the underlying economic activity. Moreover, operations CF are the core of the DCF model. These elements explain the focus on current approaches known to model operations CF.

2.1. The Indirect Method

The Indirect Method (IM) for modelling operations CF is a compulsory part of textbooks related to corporate finance and is a standard in practice - Krishnan and Largay (2000) report that 97% to 98% of companies use the IM. Brahmasrene et al. (2004) confirm that 82% of executives, CFOs, and managers also use the IM. This method entails calculating operations' CF indirectly - from the earnings restated from accruals, i.e. the EBIDTA, as well as from the variation of Working Capital (WC) (e.g. Damodaran 2010;

Ross et al. 2015; Vernimmen et al. 2017), which specifically materializes the effects of the payment terms, inventory, and production cycle lengths. Then and for a given period, operating CF is calculated as follows:

$$Operating\ CF = EBITDA - \Delta WC \tag{1}$$

The modeling of EBITDA does not present any particular difficulties. However, WC and its variations (noted ΔWC) are more problematic. Receivables, inventories, and payables that make up WC are modeled using specific financial ratios (e.g., Ross et al. 2015; Vernimmen et al. 2017), as summarized here:

$$Receivables(forecast) = Turnover(forecast).\left[\frac{Receivables(actual)}{Turnover(actual)}\right] \tag{2}$$

In this example, Receivables(forecast) are modeled by multiplying the Turnover(forecast) with a specific ratio (in brackets) based on actual data to convert the turnover in receivables. The same principle is applied for other WC components (e.g., payables, inventories). This model makes sense up to considering the *tautology* that appears by ceasing to distinguish between actual and forecasted data, in Equation (2):

$$Receivables = Turnover.\frac{Receivables}{Turnover} = Receivables \tag{3}$$

This fact reveal some limits regarding the possibility to go deeper in the mechanics of CF – which is precisely the question here. The second issue concerns the implicit hypothesis regarding the *constancy* of such financial ratios (McLeay and Trigueiros 2002), as quantity of studies report a marked tendency to evolve over time (Barnes 1987; Tippett 1990; Tippett and Whittington 1995; Sudarsanam and Taffler 1995; Whittington and Tippett 1999; Ioannidis et al. 2003; Peel et al. 2004; McLeay and Stevenson 2009). Studies also report an unexpected skewed statistical distribution for these financial ratios (Deakin 1976; Frecka and Hopwood 1983; Karels and Prakash 1987; Tippett 1990; Tippett and Whittington 1995; Whittington and Tippett 1999). This contrast with the usual conceptions indicates the lack of mastery of these ratios and directly challenges their use for prospective purposes, as emphasized by McLeay and Stevenson (2009) - which is precisely the case with the IM. Whittington (1980) and McLeay and Trigueiros (2002) insist on the limits of simple ratios regarding the difficulty to combine any economic activity, prices and volumes in evolution (e.g., growth, seasonality) with various lag effects (e.g., payment terms, inventories). These different elements, combined with the IM’s ubiquity, fit well with the poor quality of CF forecasts and further explain their "*naive*" resemblance to earnings forecasts (Givoly et al. 2009; Pae and Yoon 2012). As the IM is closer to earnings than to CF, it is more of an ersatz than a solid foundation for DCF. Our objective is to go beyond the current formulation of CF (Equation 1) by avoiding the use of financial ratios (Equation 2), assumed to be constant, and the tautological relationships they imply (Equation 3).

2.2. The analytical accounting stream of research

The search for theoretical foundations and a mathematical language underlying accounting peaked between the 1950s and 1970s (Gaffikin 2003, 2005). The research carried out by Mattessich (1958), Ijiri (1965, 1967, 1971, 1975), Tippett (1978), Carlson and Lamb (1981), and Balzer and Mattessich (1991) in particular is illustrative of this quest for axiomatic systems allowing to capture the essence of accounting language and allowing its mathematical formalization. This search for a mathematical formulation of accounting, to describe the temporal evolution of a balance sheet and its components, precedes our work. The research carried out in particular by Mattessich (1957, 1964), Williams and Griffin (1964), Ijiri (1965), Corcoran (1968), Butterworth (1972), Carlson and Lamb (1981), Ellerman (1985), Leech (1986), Arya et al. (1996), Cooke and Tippett (2000), Girardi et al. (2011), and Gentili and Giacomello (2017) - this research is clearly aimed at providing accounting with a specific matrix computational structure. Our study shares the same objectives and therefore continues the efforts of this research stream, but differs from it in the mathematical means used to formulate the accounting language. The major difference lies in our choice of modeling accounting at the most granular level of accounting entries, whose complexity (Leech 1986 ; Girardi et al. 2011) is currently avoided by using the current matrix algebra : "*the matrix framework is far simpler than the quantitative management of accounts with opposing entries, the so-called "T accounts*"" (Girardi et al. 2011, p. 3). In this formalism, the balance of the various accounts of a balance sheet at a time "t" is represented as a vector - the value of the balance of the balance sheet accounts at period "t+1" being obtained by multiplying the first vector with a matrix whose coefficients allow the amounts to circulate between the different accounts. Thus, the evolution of the balance of accounts of a balance sheet is made possible by iterating these matrix products on the vector describing the initial balance sheet.

Our paper does not rely on this matrix formalism for several reasons. The first point concerns the status of mathematical objects and of matrix coefficients in particular. These coefficients play a central role in the evolution of balance sheets, however these objects do not find any obvious correspondence with any real accounting concept. This raises the question of the possibility to describe reality without referring to its own objects. The status of the coefficients raises a more practical difficulty regarding their concrete use. These coefficients act as substitutes for accounting entries, as fundamental as universal in the description of the most varied economic operations and in the dynamics of balance sheets, but whose modeling is considered too complex to be implemented (Leech 1986 ; Girardi et al. 2011). However, the use of these substitutes has clearly not eliminated the complexity of the models, as explained by Gentili and Giacomello (2017, p. 28) : "*the evolution of the balance sheet depends on a huge number of variables especially when the number of time steps n increases. Naturally this is a trait that can't be avoided by any formulation.*" Indeed, the free evolution of the accounts of a balance sheet implies to allow the coefficients to evolve themselves in the course of time, explaining the profusion of parameters that are hard to accommodate with the real world. In spite of the difficulties posed by this matrix formalism, the idea to describe the fundamental principles (axioms) of the accounting language and to give it a mathematical formulation remains relevant. Our approach is distinctive in its commitment to faithfully reflect the reality of accounting entries, so as to deal upstream with their supposed complexity (Leech 1986 ; Girardi et al. 2011), to allow the description of any economic activity and to produce corresponding flows of accounting information. We will use this foundation to measure and study in-depth the mechanisms of value creation.

3. A mathematical theory of accounting

This section presents the general framework of our theory and defines a set of axioms, from which we contribute a mathematical form to the accounting language. Based on this mathematic-accounting language, we provide a general description for any economic activity, in form of a balance sheet, CF, and DCF evolving over time. In this respect, we specify that our theory is described in a deterministic framework, which finds several justifications here. First of all, our theory aims at describing as faithfully as possible the accounting practice, which describes for the most part real and observed facts. Adding assumptions as to the stochastic nature of the phenomena would take us away from the reality of accounting practice, without any real and serious motivation. The alignment of accounting norms (e.g., IFRS and U.S. GAAP) with the "actuarial/forward-looking" view carried by the DCF model (Cardao-Pito and Ferreira 2018 ; Markarian 2018) does not justify any departure from the deterministic framework, since the standard version of DCF is itself described in a deterministic framework. This framework also reveals our contribution in simple terms, which stochastic assumptions would tend to hide without clear necessity.

3.1. Framework and units of analysis

Definition 1 A *company* is considered as a portfolio of *economic activities*, of various types, evolving over time and defined below.

Definition 2 An *economic activity* (noted "A") is a series of specific *economic operations* (a_n) achieved per period (indexed with n) forming a vector noted $|a_1, a_2, \dots, a_{t-1}, a_t|$.

Definition 3 An *economic operation* is a basic and specific economic transaction, ranging from basic (e.g., purchase, sale, service, work) to complex forms (e.g., project, investment).

These definitions require an accounting description of the economic operations under consideration and of the effects of their repetition over time - effects that are naturally specific to each type of activity.

3.2. Accounting axioms

We state describe the core features of our theory of accounting, as a language that can describe economic *operations*, which form the *activities* conducted by a company. We adopt the three axioms of Ijiri (1965), given in a resumed to capture some general accounting features of the notions defined in previous section:

Axiom 1 (Ownership). This axiom involves four factors: a subject, objects, a relation called ownership, and time. This defines the ownership of companies (subject) with respect to the content of their activities (objects), in evolution over time (Section 3.1).

Axiom 2 (Quantities). Accounting deals with quantifiable objects, based on contingent measures, for example number of units sold, volumes, and weights. This defines the way to measure economic operations and activities (Section 3.1).

Axiom 3 (Exchanges). A subject's decreased ownership of an object (e.g. a good) is balanced by an increased ownership of another object (e.g. money). The axiom of exchanges describes the nature of what accounting should measure. Ijiri (1965, p 42) proposes the notion of "uniform measure" or "u-measure" to give an accounting description to simple economic operations. Ijiri (p 46, 47) describes the general contours of this value measurement function, which nevertheless remains lacking of an explicit formulation. To address this, we introduce a fourth axiom.

Axiom 4 (Measures). Accounting makes it possible to describe the impact of any single economic operation, using successive and specific accounting entries. These entries can be synthesized using the notion of scheme, noted "S", which describes the dynamic-accounting footprint of a single economic operation in a related balance sheet's accounts. A scheme provides a complete accounting picture of a single economic operation over time (e.g., a single purchase or sale). Different schemes can be combined to describe the complete cycle of an economic operation (e.g., purchase, sale), which often involves the use of different resources (e.g., materials, labor), until completions. We specify that schemes are described at a monthly frequency, which is a standard frequency for the production of accounting by companies. Also, schemes can be truncated after the last accounting entry, when accounting balances are stabilized.

Three examples illustrate the concept of a scheme and its ability to describe the complete cycle of an economic operation. Table 1 illustrates a scheme related to an investment and the collection on a single non-performing loan portfolio. This portfolio is acquired at 100.0 thousand dollars, generates a variable collection and collection costs for six months, paid cash. Its book value is estimated and amortized from the value of future CF (net of collection costs), discounted at the internal rate of return (7,26%).

Table 1. Accounting scheme related to a loan portfolio - in thousand dollars

Items	Accounts	0	1	2	3	4	5	6
Assets	Loan value	100.0	82.3	52.2	28.0	11.1	1.9	0.0
Assets	Cash position	-100.0	-75.0	-39.0	-11.0	8.0	18.0	20.0
Total Assets		**0.0**	**7.3**	**13.2**	**17.0**	**19.1**	**19.9**	**20.0**
Equity	Collection (earnings)	0.0	30.0	70.0	100.0	120.0	130.0	132.0
Equity	Coll. costs (earnings)	0.0	-5.0	-9.0	-11.0	-12.0	-12.0	-12.0
Equity	Amortization (earnings)	0.0	-17.7	-47.8	-72.0	-88.9	-98.1	-100.0
Total Equity + Liabilities		**0.0**	**7.3**	**13.2**	**17.0**	**19.1**	**19.9**	**20.0**

Notes. This scheme consists of five account vectors, of which for example the one of cash position equal to $|-100.0, -75.0, -39.0, -11.0, 8.0, 18.0, 20.0|$. This vector is arbitrarily truncated over 7 periods, with a balance of 20 implicitly assumed to continue for subsequent periods - as for the other accounts.

Table 2 illustrates the scheme related to a financial loan (100 thousand dollars) granted to a customer for a period of 6 months, at a rate of 1% per month. The principal is amortized on a linear basis. This scheme describes the transaction from the lender's side.

Table 2. Scheme related to a single financial loan

Items	**Accounts**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Assets	Loan	100.0	83.3	66.7	50.0	33.3	16.7	0.0
Assets	Cash position	-100.0	-82.3	-64.8	-47.5	-30.3	-13.3	3.5
Total Assets		**0.0**	**1.0**	**1.8**	**2.5**	**3.0**	**3.3**	**3.5**
Equity	Interests (earnings)	0.0	1.0	1.8	2.5	3.0	3.3	3.5
Total Equity + Liabilities		**0.0**	**1.0**	**1.8**	**2.5**	**3.0**	**3.3**	**3.5**

Notes. This scheme consists of three account vectors, of which for example the one of cash position equal to $|-100, -82.3, -64.8, -47.5, -30.3, -13.3,\ 3.5|$. This vector is arbitrarily truncated over 7 periods, with balance of 3.5 thousand dollars implicitly assumed to continue in the next periods - as for the other accounts.

Table 3 illustrates the scheme related to a commercial activity : a single product is bought at 85 dollars per unit from the supplier and paid with a delay of one month. A VAT of 10% is applied on this purchase and is repaid the month following the purchase. The inventory is justified by a sale one month after the purchase, at a unit price of 100 dollars, paid within two months.

Table 3. Scheme related to a commercial operation

Ass./Liab.	**Accounts**	**1**	**2**	**3**	**4**	**5**	**6**
Asset	Inventory asset	85	0	0	0	0	0
Asset	Deductible VAT	8,5	0	0	0	0	0
Asset	Receivables	0	110	110	0	0	0
Asset	Cash (Sale + Purchase)	0	-85	-95	15	15	15
Total Assets		**93,5**	**25**	**15**	**15**	**15**	**15**
Equity	Sales (earnings)	0	100	100	100	100	100
Equity	Accrued purchase (earnings)	85	0	0	0	0	0
Equity	Purchase (earnings)	-85	-85	-85	-85	-85	-85
Liabilities	Payables	93,5	0	0	0	0	0
Liabilities	Collected VAT	0	10	0	0	0	0
Total Equity + Liabilities		**93,5**	**25**	**15**	**15**	**15**	**15**

Notes. This scheme consists of nine account vectors, of which for example the one of cash position equal to $|0.0, -85.0, -95.0,\ 15.0,\ 15.0,\ 15.0|$. This vector is arbitrarily truncated over 6 periods, with a balance of 15 implicitly assumed to continue in the next periods - as for the other accounts.

In mathematical terms, a scheme can be shown as a group of vectors - each describing the evolution of the balance of accounts involved in the description of the same economic transaction, until the last accounting entry is completed. This axiom and the concept of scheme allow to represent in simple mathematical terms the infinite variety of accounting entries that can be used to describe any operation. In doing so, this simple and general formalism solves the difficulty, expressed by Leech (1986) and Girardi et al. (2011), in modeling accounting at the most fundamental level of accounting entries. The first corollary of this axiom is that all the operations in the same economic activity can be described by the same scheme. The second corollary is that these schemes are supposed to remain constant over time. This assumption, which could be considered as a constraint, is relaxed in a later section.

3.3. A mathematical formulation of accounting

We present how to give a complete accounting description to a contingent economic activity (A). To do this, we use the notion of accounting scheme (S) to describe each of the economic operations that make up the economic activity (A). We prove (Appendix A.1) that the combination of S and A produces an information flow describing the evolution of the balance of all the accounting accounts mobilized by this activity. We prove (Appendix A.1) that the combination of S and A takes two equivalent mathematical forms, which we will use depending on our needs:

- A product between a "Toeplitz" matrix (see Poularikas 1999), formed by S, and A.
- A "discrete convolution product" (DCP, see Gray 2006), associating S and A.

We express these equivalent mathematical operations through a "bracket" notation:

$$\text{Accounting balances} = \langle S|A\rangle \tag{4}$$

The balance sheet computed by Equation (4) is in the form of a set of vectors, of identical size to A and shares the same accounts as S. Appendix A.1 provides a concrete illustration of the calculation of the evolution of cash positions and continues to the complete balance sheet calculation. Our formulation of a balance sheet differs markedly from those proposed by the analytical accounting stream of research, discussed in Section 2.2, since our calculations deal directly with economic operations and their translation into fundamental accounting entries, synthesized in the form of schemes - a granular level of operation and accounting entries that has not been explored until now, because considered too complex (Leech 1986 ;

Girardi et al. 2011). Our theory describes economic activities at a more fundamental level than the approaches proposed so far that remain at the level of accounting aggregates. The IM, presented in Section 2.1, also operates at the level of accounting aggregates at which the effects of payment and storage delays are approximated - a level that does not accurately describe the mechanisms underlying CF. For this reason, we propose to formulate CF directly from the level of fundamental economic operations.

4. Reducing CF and DCF to accounting

This section introduces a new formulation to the notions of CF and DCF, directly from our mathematical theory of accounting. This study attempts to reduce and unify finance and accounting, and aims to produce a more general and fundamental formulation of the DCF model.

4.1. A reformulation of CF

We calculate the evolution of CF of any activity directly from the evolution of the cash balances contained in balance sheets. For simplicity, we reduce the notion of scheme to its single cash dimension - its status of group of vectors is therefore reduced to that of a single vector. To shift from cash positions to CF, we simply derivate over time (operation noted "Δ") the cash balance vector S provided by Equation (4):

$$\mathrm{CF} = \Delta\langle \mathrm{S}|\mathrm{A}\rangle \tag{5}$$

With $\langle \mathrm{S}|\mathrm{A}\rangle = |c_1, c_2, \dots, c_{t-1}, c_t|$ - "c_t" components being equal to cash balance over time and with $\Delta\langle \mathrm{S}|\mathrm{A}\rangle = |c_1, c_2 - c_1, \dots, c_{t-1} - c_{t-2}, c_t - c_{t-1}|$ being equal to a stream of CF. The distributive properties within a DCP (e.g., Bracewell 1965), makes it possible to reformulate CF:

$$\mathrm{CF} = \Delta\langle \mathrm{S}|\mathrm{A}\rangle = \langle \Delta\mathrm{S}|\mathrm{A}\rangle = \langle \mathrm{S}|\Delta\mathrm{A}\rangle \tag{6}$$

Among these possible formulations of CF, $\langle \Delta\mathrm{S}|\mathrm{A}\rangle$ appears as the most intuitive, as $\Delta\mathrm{S}$ is equal to the CF vector that is related to a single economic operation. Our formulation of CF encompasses the notion of *Free Cash Flow*, used in the DCF model, which is equal to operating cash flow minus cash flow from investments. Indeed, two of our examples (Table 1 and 2, Section 3.2) show how schemes makes it possible to integrate investments directly into operating flows, when the former are directly part of the operating cycle (an investment is associated with a given economic operation). It is still possible to envisage that investment are not part of the operating cycle (an investment is associated with a given activity) - this situation would imply separate schemes and activities (operating and investment). Therefore, our definition of CF can be used as a natural substitute for that of Free Cash Flow, which is why we will use it to progressively reformulate the DCF model.

4.2. A reformulation of Future Values

The Future Value (FV) is equal to the sum of future CF capitalized at a given interest rate. This notion is an intermediate step toward DCF models and is currently formulated as follows:

$$FV = \sum_{i=1}^{t} CF_i . (1+r)^{t-i} \quad (7)$$

This standard formulation of the FV combines any sequence of CF (indexed in time by "i") with the effects of capitalization, induced by the interest rate (noted "r"). This standard formulation is both reduced and generalized (proof in Appendix A.2) through our formulation of CF, in which we add the effects of interest capitalization:

$$FV = \sum \langle \Delta_2 \mathrm{S} | \mathrm{A} | \mathrm{F} \rangle \quad (8)$$

This reformulation requires some explanation of the notations. First, we set aside the summation index because of the implicit length of the activity "A". Also, $\Delta_2\mathrm{S}$ refers to the second derivative of the cash balance vector (S) relative to time. Finally, the new vector "F" consists of the same capitalization factors, as in the standard formulation of the FV: F = $|(1+r)^1, (1+r)^2, \dots, (1+r)^{t-1}, (1+r)^t|$. Thus, Equation (8) generalizes the standard formulation of FV and reduces it again to accounting mechanisms, inscribed in form of S. Before proceeding to the reformulation of DCF, we illustrate the generalization capacity of our formalism by integrating the effects of inflation using the same process used from the CF, in Equation (6), to the FV, in Equation (8):

$$FV = \sum \langle \Delta_3 \mathrm{S} | \mathrm{A} | \mathrm{F} | \mathrm{P} \rangle \quad (9)$$

This formulation leads us to a third derivation of S ($\Delta_3 S$), which is made necessary to embed the effects of inflation through a new vector: P = $|(1+i)^1, (1+i)^2, \dots, (1+i)^{t-1}, (1+i)^t|$ where "i" acts as the rate of inflation.

4.3. A reformulation of DCF

The previous steps almost completed the process, from a theory and mathematical formulation of the accounting to the reformulation of DCF in more fundamental and general terms. This is particularly evident given that the DCF is simply equal to the present value of FV:

$$DCF = (1 + r)^{-t}.FV \tag{10}$$

We reformulate the DCF by injecting our formulation of FV (Equation 9) into the Equation (10):

$$DCF = (1 + r)^{-t}.\sum\langle \Delta_3 S|A|F|P\rangle \tag{11}$$

To summarize the path taken so far, our approach involves reducing the notions of DCF into fundamental accounting terms, thus describing the underlying operations and economic activities. As such, our unification of accounting and finance paves the way to generalize and to explore the phenomenon further.

5. A dynamic perspective on CF, FV and DCF

We describe and study the mechanisms through which the dynamic-accounting footprint of schemes (S) determines the general pattern of the behavior of CF and DCF, in form of typologies and dynamic limits. This knowledge is decisive in the succeeding section, where we explore the means of explaining and controlling value creation mechanisms and stock returns.

5.1. A study of CF behavior and limits

We study the dynamic behavior of CF, in Equation (6), as a product of the combination between the dynamics of the activity (A) and its scheme (S), which has a variety of forms, suggesting a field of possibilities. To the best of our knowledge, the only known result regarding the dynamic behavior of CF was established by Hamman (1996), Churchill and Mullins (2001), and Dreyer et al. (2013) by modeling a growth limit, named "Cash Flow Sustainable Growth Rate" (CFSGR), beyond which CF would cross into negative territory. The lack of attention paid to CF explains the limited impact from these studies; however, their importance is implied by the place of CF in DCF. Nevertheless, CFSGR models are based on the IM, which suggest the possibility of deepening knowledge regarding CF behavior (Section 2.1). We formulate the CFSGR phenomenon by imposing on the CF in Equation (6) to be null at each period, under the effect of a constant growth "g" in the economic activity (A) - proof in Appendix A.3:

$$\sum_{i=1}^{X} \Delta s_i.(1+g)^{-i} = 0, \quad (12)$$

Where "Δs_i" refers to the "X" components of ΔS. In Equation (12), $\sum_{i=1}^{X} \Delta s_i.(1+g)^{-i}$ can be interpreted as a unit CF function that corresponds to the CF produced by each economic operation - comprised in an economic activity (A) evolving at a constant rate of growth.

Our reformulation of the CFSGR leads us to a more fundamental and general understanding of the dynamic behavior of CF, that is, beyond an important but specific growth limit. Indeed, this unit CF function takes the form of a polynomial whose number of solutions depends directly on the number of sign changes of ΔS, based on the famous "Descartes rule". Therefore, the condition of the CFSGR existence is not restrictive. For simplicity, we limit ourselves to studying the solution closest to zero in Equation (12), which we name "λ" and it suffices to depict the behavior of CF at "usual" growth regimes. Aside from λ, the unit CF function presents a second remarkable value for a null growth, in which the sum of ΔS components is necessarily equal to the unit margin (earnings) noted "M". Indeed, the notions of cash and earnings are supposed to converge at the end of all operations, which S is supposed to reflect. We represent the local behavior of the unit CF function (for a growth "g" ranging between 0 and λ) with a linear interpolation:

$$Unit\ CF = M.\left(1 - \frac{g}{\lambda}\right) \quad (13)$$

This unit CF function reveals the different typologies of CF behavior, of which the scheme turns out to be the fundamental determinant. When M and λ are positive, CF behave as predicted by Hamman (1996), Churchill and Mullins (2001), and Dreyer et al. (2013), decreasing with growth until they become negative, beyond λ. We assume that most activities are in this typology, specifically, that λ is positive, given that companies tend to pay their suppliers before being paid by their customers. When M is positive and λ is negative, CF increase with growth - this possibility was not considered in the research on the CFSGR; however, it also applies to firms that manage to collect cash from customers before paying their suppliers. We do not elaborate on the last two typologies ("M" negative), for obvious reasons of economic viability.

Our analysis emphasizes the role of schemes regarding to the impacts of growth on CF at the level of a single operation and at the level of the whole activity. It is key to note that growth erodes CF for a single operation and that it can increase CF for a whole activity, due to the compound effects of growth on business volume, which can supersede the negative effects of growth regarding λ. The notion of FV makes it possible to integrate both levels of CF in the long-term and to integrate the compound effects of financing and inflation in parallel of growth. Their relative effects is of interest and pave the way to the study of DCF.

5.2. A study of FV behavior and limits

We develop the implications discussed in Section 5.1 to describe the dynamic behavior of the FV. The formal proximity between our CF (Equation 6) and FV (Equation 9) models underlines the nature and central role of S in both models. The specificity of the FV lies in the introduction of the effects of financing and inflation alongside those of growth - on purpose, we specify the form of the related vectors:

$$\mathrm{A} = Q.\,|\gamma^1, \gamma^2, \ldots, \gamma^{t-1}, \gamma^t|, \text{ with } \gamma = 1 + g \text{ and "g" as a rate of growth.} \tag{14}$$

$$\mathrm{F} = \quad |\alpha^1, \alpha^2, \ldots, \alpha^{t-1}, \alpha^t|, \text{ with } \alpha = 1 + r \text{ and "r" as a rate of financing.} \tag{15}$$

$$\mathrm{P} = \quad |\pi^1, \pi^2, \ldots, \pi^{t-1}, \pi^t|, \text{ with } \pi = 1 + i \text{ and "i" as a rate of inflation.} \tag{16}$$

To study stylized facts, as in Section 5.1, we set growth, financing, and inflation rates as constants. The parameter "Q" (Equation 14), represents the initial level of activity (economic operations) and is an exception to the other vectors. However, Q can be factored out of the bracket of Equation (9) to establish a formal symmetry in the composition of A, F, and P. Further, the commutativity of DCP (e.g., Bracewell 1965) indicates that this apparent symmetry is more fundamental, as these vectors can be swapped without changing the outcome. This symmetry permit to conclude that growth (g), financing (r), and inflation (i) rates are perfectly symmetrical within FV. In other words, 1% of growth, for example, will have the same effect on FV, as 1% of financing or inflation. The corollary of this symmetry is that λ (Section 5.1) unifies different financial limit, known separately to the domain of growth, the financing rate and inflation - to our knowledge, this is the first mention of such a limit in the literature.

The theoretical and practical implications of these symmetries are evaluated through a concrete example that measures the FV generated by an economic activity projected to a certain investment horizon. The example in question is that of a loan portfolio, introduced in Section 3.2 (Table 1), whose scheme (S) is reduced here to the dimension of cash (in thousands dollars) :

$\mathrm{S} = |-100.0, -75.0, -39.0, -11.0, 8.0, 18.0, 20.0|$.

S is characterized in particular by its unit margin M (20.0k$) and its dynamic limit λ (7.3%). The stylized facts about FV are highlighted from an activity "A" consisting of the purchase of a single loan portfolio in the first period (a_1=1), renewed in each new period following a constant growth rate :

A = $|1.0, 1.02, 1.04, 1.06, 1.08, 1.10, 1.13, 1.15, 1.17, 1.20, 1.22, 1.24|$, with g = 2% for the illustration.

These elements allow first to calculate a series of CF :

$\mathrm{CF} = \langle \Delta_1 \mathrm{S}|\mathrm{A}\rangle = |-100.0, -77.0, -42.5, -15.4, 3.3, 13.4, 15.6, 15.9, 16.3, 16.6, 16.9, 17.3|$.

This series of CF is limited to 12 months, in line with the current projection horizon of investors in financial markets (see Section 6.1). This horizon reinforces the short term erosive effect of growth, visible at the completion of the first scheme (7 months) with CF reaching 15.6k$ - significantly below the 20.0k$ of the unit margin. From 7 to 12 months, CF increase and finally reach 17.3k$, due to the composition of growth but still below the unit margin as a short-term weight. Indeed, it takes 20 months to generate CF superior to 20k$. That said, the effects of financing (r = 2%) are integrated with CF to measure the FV[1]:

$\mathrm{FV} = \sum\langle \Delta_2 \mathrm{S}|\mathrm{A}|\mathrm{F}\rangle = \sum|-100.0, -79.0, -46.1, -19.9, -1.6, 8.4, 10.9, 11.4, 11.9, 12.5, 13.1, 13.7|$.

The FV is equal to the sum of a stream of CF, equal to -164.7k$. Beyond this result, we introduce the notion of "unit FV" to identify and measure the background effects of the investment horizon on the *perception* of value creation initiated by a single economic operation, alongside all the other factors in the vector $\langle \Delta_2 \mathrm{S}|\mathrm{A}|\mathrm{F}\rangle$ - in our example the unit FV reaches \$13.7k at the end of the 12-month horizon. This notion allows to measure a slice of the total FV at a specific investment horizon. It follows that the sum of the unit

[1] Inflation is not integrated for simplicity, but the symmetry with growth allows to consider its effects.

FV over the totality of the investment horizons (from 1 to 12 months in the example here) and multiplied by the number of economic operations realized during the first period equals to the total FV.

Figure 1 illustrates the notion of unit FV with our example of a loan portfolio, as a function of growth and financing rates and investment horizon. Two investment horizons (12 and 24 months) illustrate the evolution of the unit FV and its symmetrical increase toward the λ curve, due to the volume effect on activity and the composition of interests, both beneficial over the time.

Figure 1. Unit FV for an investment horizon of 12 months (left) and 24 months (right) - contour lines in thousand dollars and λ curve (null value) in red.

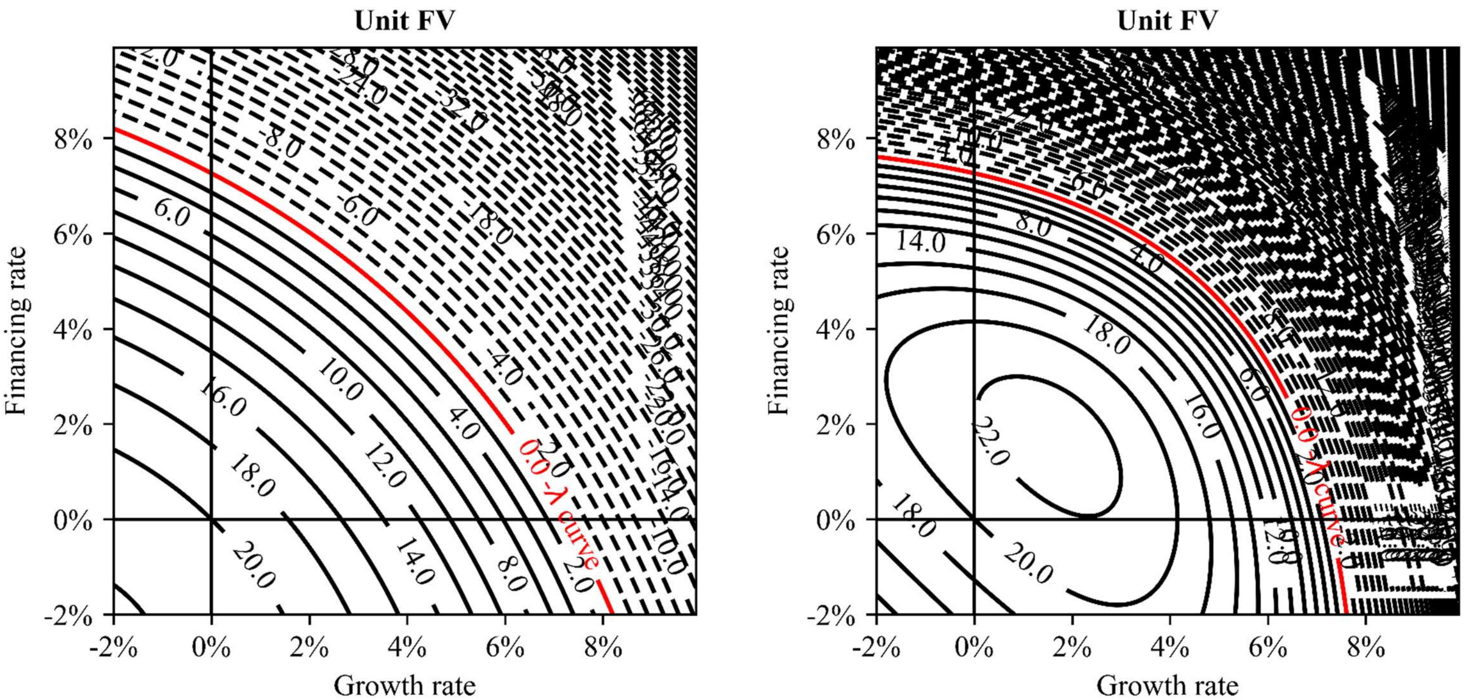


Figure 1 illustrates the roles of M and λ as constants across investment horizons. Meanwhile, symmetries remain and evolve with the composition of growth and financing on the unit FV - an increase in value perceived specifically at each investment horizon. That being said, the shape of S is critical to the level of M and λ, so Section 6.1 will discuss their relationship to the value creation actually measured in the financial markets. Before looking at this point in more detail, it remains to see the DCF model and its important specificities regarding the perception of value.

5.3. A study of DCF behavior and limits

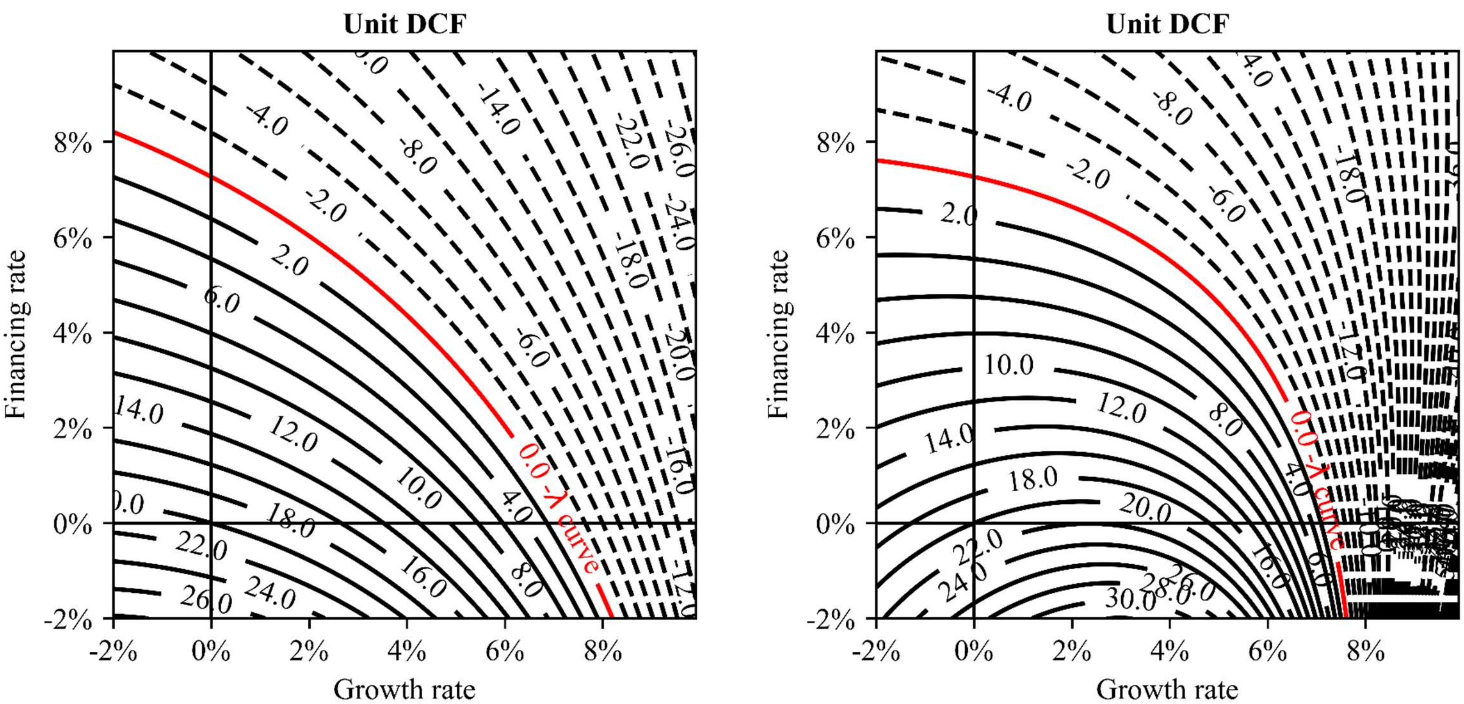
We develop here the implications of Section 5.2, as to the stylized facts relevant to describe the dynamic behavior and limits of DCF. First, we highlight that DCF is constructed on a discounted FV (Section 4.3). Regarding the symmetries identified in FV (Section 5.2), the adjunction of the discount factor $(1+r)^{-t}$ introduces a *symmetry breaking* between the effects of financing on the one hand and those of growth and inflation on the other. In other words, 1% of the rate of financing no longer produces the same effects as 1% of the rate of growth or inflation in DCF. This discount factor thus exerts negative pressure on DCF in the single dimension of the rate of financing, without equivalence in the domains of growth and inflation. We analyze the consequences of this symmetry breaking for the possible dynamic limits of DCF. Thus, as with CF and the FV, we look for conditions under which the DCF can be cancelled:

$$DCF = (1+r)^{-t}.\sum\langle \Delta_3 \mathrm{S}|\mathrm{A}|\mathrm{F}|\mathrm{P}\rangle = 0 \tag{17}$$

The discount factor $(1+r)^{-t}$ plays no role in the solutions of Equation (17), which remain the same as for FV and are equal to λ (Sections 5.1 and 5.2). This significant result maintains the identity of the three dynamic-financial limits for DCF, despite the symmetry breaking coming from the discount factor. This result allows us to prove the identity of three limits, which until now were considered separately and formulated in distinct terms. The first of these limits is the well-known IRR, which corresponds to the rate of financing cancelling the value of DCF. The second limit is the CFSGR, already discussed in Section 5.1 and modeled by Hamman (1996), Churchill and Mullins (2001), and Dreyer et al. (2013) as the rate of growth canceling the CF, and the related consequences had not been derived until the domain of DCF. The third limit is expressed in the domain of inflation and is capable of cancelling both CF and DCF - which we name "cash-flow sustainable inflation rate", as we have not found it in the literature. To the best of our knowledge, our work establishes for the first time the unity of these three limits.

As in Sections 5.1 and 5.2, the theoretical and practical implications of these symmetries are evaluated through a concrete example, allowing to measure the value (DCF) generated by each economic operation (unit DCF), projected at a certain horizon in the future. In our previous example (Section 5.2) the unit FV reaches \$13.7k at a 12 month horizon – the discounting of this value gives a unit DCF equal to 10.8k\$ at the same horizon. Figure 2 thus represents the levels of unit DCF of our example of loan portfolio, as a function of any rates of growth and financing.

Figure 2. Unit DCF for an investment of 12 months (left) and 24 months (right) - contour lines in thousand dollars and λ curve (null value) in red.



The symmetry break between the erosive effects of growth and financing are visible at a 12-month horizon and at a 24-month horizon the beneficial effects of growth become visibly stronger over time on CF and DCF. These observations suggest that we should be interested in the real horizon of investment in financial markets, in order to verify whether this horizon exerts a *background* effect making the investors more sensitive to short-term erosive effects, or whether they are more sensitive to positive compositional effects, rather visible in the longer term.

The effects of growth, financing and inflation are erosive, or compositional depending on the projection horizon of investors in financial markets. The effects of the investment horizon are evaluated in more general terms in Appendix A.4 and these effects provide a reasonable explanation as to the same effects on stock returns, discussed in the next section. Meanwhile, the core of value creation fundamentaly lies in the companies' ability to generate CF and value. The next section thus confronts our theory with actual observations regarding value creation by companies (stock return) and its sensitivity to different market condition factors (e.g., growth, financing and inflation).

6. Contributions to practice and research

The literature is analyzed around the role played by each of the factors involved in the measurement of the DCF, so as to verify the link between the predictions of the theory developed here and the observations on value creation measured on financial markets (stock returns), in relation to all the factors involved in the DCF model. The point of this lies in the possibility of explaining and unifying different stylized facts regarding stock returns, through the mechanisms of value creation described above. These stylized facts relate in the first place to the sensitivity of stock prices (stock return) to the *internal* factors affecting the very heart of the value creation, i.e. CF, S naturally, as well as the horizon of investors in financial markets. This analysis will be complemented by the analysis of the link between stock returns and various market factors whose erosive effects, visible in the studies, fit well with the short term horizon of investors in financial markets, just discussed.

6.1. Internal factors

We discuss the empirical links between S, CF, DCF and value to stock prices and returns, with the objective of verifying whether the value creation factors, in the hands of the firms, do indeed exert a material and visible influence given the horizon of investors in financial markets.

These different notions have quite complex relationships, as the conception of value and its actual measurement by economic actors have evolved significantly over time. Rappaport (2005) follows the thread of this historical evolution, during which the use of the DCF model has apparently evolved, in parallel with the significant shortening of the horizon of investors in financial markets, as we will see. We propose to follow this evolution to clarify the contribution of our theory to the current mechanisms of formation and perception of value by economic agents. Rappaport highlights the central role of the DCF model as a fundamental measure of the value of assets and companies' stocks, in particular. The notion of *fundamental efficiency* thus designates the idea that asset prices must necessarily converge towards the fundamental value produced by the DCF model. This idea is widely shared by professionals, academics, and studies confirm its empirical relevance (e.g., Kaplan and Ruback 1995, 1996). This finding confirms the possibility that our theory may deepen the measurement of value and thus explain stock prices on financial markets. Though, the implementation of DCF raises deep issues, as most investment professionals consider distant CF projections as "too time-consuming, costly, and speculative to be useful" (Rappaport 2005, p65). These difficulties explain why the measurement of absolute values has been put aside for the measurement of their variations and therefore the focus changed from estimating stock prices to estimating stock returns. The notion of *informational efficiency*, also discussed by Rappaport, covers this new logic consisting of integrating all available information to anticipate stock returns, justified by a change of the value of a stock and its main underlying factors, *future* CF. Studies confirm the close ties between CF forecasts and stock returns (e.g., Vuolteenaho 2002 ; French 2008 ; Larrain and Yogo 2008 ; Jansen 2021).

Meanwhile, the shift from the notion of fundamental efficiency to the informational efficiency has also been accompanied by a significant shortening of the investment horizon on financial markets. Rappaport points out that the investment horizon of financial investors has shortened significantly from seven years in the 1960s, to one year. This shortening of the horizon is still attested by studies on momentum strategies reveal that profitable investment horizon does not exceed one year in the US financial markets (e.g., Levy 1967 ; Jegadeesh and Titman 1993 ; Griffin et al. 2003 ; Asness et al. 2013). In Chinese financial markets, Gang et al. (2019) observe a strong reactivity of investors to CF news, with profitable investment horizon inferior to one week. Chen et al. (2013) nuances these observations by showing that S&P500 stock returns are strongly related to CF news, whose importance increases with the investment horizon, especially for horizons over two years. Studies confirm the link between *real* CF and stock return (e.g., Da 2009 ; Hou et al. 2011 ; Novy-Marx 2013 ; Ball et al. 2016 ; Foerster et al. 2017).

To come more specifically to our theory, this shortening of the investment horizon on financial markets is significant in two ways. First of all, a short-term investment horizon is supposed to have a significant influence on the perception of the value creation by investors (see Section 5.2 and 5.3), which we will come back to in the next section to explain the effect of market factors on stock returns. On the other hand, the shortening of the investment horizon provides a natural explanation for the role of *real* CF on stock returns, and it seems reasonable to assume that real CF news are used as primary information to make short-term projections of CF and DCF, whose change over time may be used to estimate stock returns. In this context, some studies establish that mastery of the lags underlying companies' real CF (e.g., payment terms, stockage) are strongly related to stock returns. In our theory, these lags are through the shape of S over the time and these lags have a strong influence on the level of λ, CF and stock returns - positive gains in payment terms allow to push λ and to create more value. In studies, the notion of Cash Conversion Cycle (CCC) measures the time elapsing between payment to suppliers and collection from customers. Zeidan and Shapir (2017) provide evidence that decrease in CCC and working capital requirements increases CF, with a significant and positive effect on the stock returns. Sorting stocks into CCC deciles, Wang (2019) finds that the excess returns of portfolios decrease almost monotonically when the CCC increases. Lin and Lin (2021) find that the aggregate CCC is a strong positive predictor of the aggregate stock market return and outperforms a series of well-known return predictors.

We provide a simple illustration of the effect predicted with our example of loan portfolio (Table 1) whose unit cash position vector follows (in thousands dollars) $|-100.0, -75.0, -39.0, -11.0, 8.0, 18.0, 20.0|$ and whose corresponding CF vector is equal to $|-100.0, 25.0, 36.0, 28.0, 19.0, 10.0, 2.0|$. We leave the initial investment unchanged (-100.0) and evaluate the effect of shortening the CCC of collecting faster the same amount of cash (120.0) through a new CF vector : $|-100.0, 40.0, 26.0, 23.0, 19.0, 10.0, 2.0|$ - that is, 15.0 more in the second period, 10.0 less in the third period and 5.0 less in the fourth period. Our shift on the unit CF vector pushes back the λ limit from 7.26% to 7.82% and therefore pushes the level of CF generated for any level of (positive) growth of the activity. Assuming a growth in economic activity by 2%, the unit CF (Section 5.1) generated by each portfolio would increase from 13.88 to 14.27, i.e., a potential increase rate of 2.76% for CF, DCF, and stock returns. In the perspective of a company manager, such evaluations should also take into account the cost of such changes in the internal process (e.g., operations, accounting).

6.2. Market factors

We begin by analyzing the effects of firms' growth on stock return and see how our accounting formulation of DCF provides an explanation. The literature has well documented the negative effect of growth in corporate assets (investments) on stock return (Titman et al. 2004 ; Fama and French 2006; Cooper et al. 2008 ; Watanabe et al. 2013 ; Cheung Jiang 2014 ; Mao, Wei 2016 ; Wen 2019). Naturally, growth in corporate assets is not necessarily tied to the growth in their activity, which we have discussed so far. Nevertheless, it is still possible to imagine a more or less direct link between these two measures of growth - this is notably the case for activities in which each economic operation requires its own investment (e.g. purchase of a debt portfolio, financial loan). Other activities may require an initial investment (e.g. plant, additional machinery) to unleash the growth potential of the activity. The relationship between investment and activity growth allows us to propose a new mechanism to complement the explanations already given for the negative relationship between investment and activity growth. Some authors (Watanabe et al. 2013 ; Cheung Jiang 2014) put forward arguments linked to the agency theory (Jensen 1986) which assume a natural tendency of managers to invest in value-destroying projects. Other authors (Titman et al. 2004 ; Fama and French 2006 ; Cooper et al. 2008 ; Wen 2019) assume a natural tendency of investors to overestimate the positive effects of investment - the subsequent downward revision of these expectations can explain negative stock returns. Our accounting formulation of DCF provides an alternative explanation for this phenomenon, since the additional growth in activity generated by investment naturally reduces the level of CF generated by each economic operation (for a positive λ - Section 5.1). The effects of this phenomenon are directly reflected in the level of DCF, justifying the decrease in firm market value. To our knowledge, Cheung Jiang (2014) is the only study that has measured the effects of activity (sales) growth on stock returns - the results show that firms with low levels of activity growth offer the highest stock returns. This supports our argument that growth erodes firms' ability to generate CF and to create value. Moreover, this study shows that activity growth destroys more value than investments, reinforcing our alternative explanation of the mechanisms leading assets and activity growth to destroy value.

Given the symmetry of the effects of growth and inflation on CF and DCF, it would seem natural that inflation would exert the same negative influence on stock returns. In this area of the literature, our prediction contradicts Fisher's theory that the expected nominal rate of return on asset is equal to expected inflation plus the real rate of return, where the ex-ante real rate of return is independent of expected inflation. This theory has given rise to numerous tests, the results of which have shown a strongly negative correlation between inflation and stock return, over investment horizons ranging from one year (e.g., Lintner 1975 ; Bodie 1976 ; Fama and Schwert 1977 ; Modigliani and Cohn 1979 ; Fama 1981 ; Boudoukh et al. 1994 ; Ritter and Warr 2002 ; Campbell and Vuolteenaho 2004) to several years (e.g., Sharpe 2002 ; Cohen et al. 2005 ; Durai and Bhaduri 2009). These results are fully consistent with our theory's predictions, especially given the investors' horizon of less than one year (see Section 6.1). On the other hand, the Fisher's theory seems to be verified in some studies and essentially for horizons greater than one year (e.g., Boudoukh and Richardson 1993 ; Solnik and Solnik 1997 ; Schotman and Schweitzer 2000 ; Kim and In 2005 ; Bhanja et al. 2012 ; Tiwari et al. 2015). The fact that inflation and stock return are positively correlated in the long run has no obvious explanation in our theory. Whatever the possible explanations for this phenomenon, it should not have any material effect on actual stock return for investors, whose horizon remains clearly short-term. This being said, the explanations given in the literature for the negative correlation between inflation and stock return are many and varied. The first explanation assumes that investors incorrectly incorporate the effects of inflation into their valuation models. This is the famous “money-illusion” hypothesis of Modigliani and Cohn (1979) that investors value stocks by discounting future CF using nominal discount rates. Ritter and Warr (2002), Campbell and Vuolteenaho (2004) and Cohen et al. (2005) produce evidence in support of the money-illusion hypothesis. Other authors (Fama and Schwert 1977 ; Boudoukh et al. 1994) find that inflation, whether anticipated or not, has a negative influence on stock returns. This suggests that inflation does have a negative influence beyond the treatment of its effects in the models. Fama (1981) brings another explanation by arguing that high levels of inflation tend to penalize real business activity, therefore real financial performances. In support of this idea, Sharpe (2002) suggests that market expectations of real earnings growth, particularly longer-term growth, are negatively related to expected inflation. Without claiming to be exhaustive of the explanations given for the negative effects of inflation on stock returns, these explanations nevertheless make it possible to underline the originality of our explanation, simple and coherent, as to the symmetry of the negative effects of growth and inflation on the level of CF, DCF and stock returns. We conclude our analysis of the effects of market factors by discussing the effects of the financing rate on stock returns. The negative effects of the financing rate on stock returns is widely documented in the literature (e.g., Bernanke and Kuttner 2005), although the phenomenon can be nuanced for periods of financial bubbles during which rate increases no longer seem to exert an effect on stock returns (e.g., Galí and Gambetti 2015). This negative influence of the financing rate is naturally explained by the erosive effect on the future CF of the DCF - justifying the decrease in values of the firms and the consequences on stock returns.

The question of the necessity of the erosive effects of market factors remains, since these effects can give way to positive composition effects, beyond a certain horizon (see Figures 1 and 2 in Section 5.2 and 5.3). The answer to this question can be found in the investment horizon on financial markets is interior of equal to 12 months (Section 6.1). Indeed, Appendix A.4 shows why it is essentially lower than the *equilibrium investment horizon*, beyond which erosive effects of market factors give way to positive composition effects. This study of the equilibrium investment horizon (Appendix A.4) thus provides a reasonable and consistent explanation for the same erosive effects observed on stock returns.

7. Conclusion

This paper contributes to the convergence between accounting and finance - particularly visible through the advent of the DCF model in the main accounting standards (e.g. IFRS, US GAAP). This evolution marks the beginning of a new "actuarial/forward-looking" phase in accounting thought (e.g., Cardao-Pito and Ferreira 2018; Markarian 2018), to which we contribute by showing that accounting can in turn advance theoretical knowledge of this core financial model and that this progress would improve its practical use.

This paper proposes a mathematical theory of accounting, as a formal language capable to simulate the flow of accounting information related to a given economic activity, but still capable of reformulating the DCF model in fundamental accounting terms, and to study value creation mechanisms as a function of its different factors. The origin of these possibilities lies in the introduction of the notion of accounting scheme (S) and its economic activity (A): the combination of S and A, with discrete convolution product (e.g., Gray 2006) produces a stream of accounting information, whose study reveals a dynamic limit to the growth of the activity (null CF), directly induced by different delays between inflows and outflows, whose effects are contained in S. This theory proves the unity of three dynamic-financial limits - intervening in the areas of growth, financing rate and inflation - beyond which economic activities generate negative CF and DCF. These three limits are more or less known in the literature (e.g., Internal Rate of Return, Cash Flow Sustainable Growth Rate), but separately and in distinct terms - their unification sheds new light on the effects of payments terms in CF, captured by accounting.

The perception of value thus depends on the shape of the scheme S (prices, payment terms and λ), market factors (growth, financing and inflation) limited by λ, but also the projection horizon of investors. The current investment horizon in financial markets, from few months to one year (see literature in Section 6.1) explains well why the effects of market factors (growth, financing and inflation) exert an erosive effect on stock prices and their returns. Indeed, a short-term investment horizon does not leave enough time for the composition of growth, interests and inflation to exert their positive influence on value, stock prices and returns. But beyond value, as perceived by investors in financial markets, this paper shows that the very heart of the value creation, by the companies, lies in the mastery of the shape of schemes, i.e. the mastery of prices and the payment terms, which make it possible to push back λ and thus create more CF, DCF and value (stock return) whatever the states of the market (growth, financing and inflation).

The deterministic nature of our theoretical framework could be considered as a limitation. This is actually the same limitation as the DCF model itself, and therefore our paper provides a deeper understanding of the standard model of value, which admittedly obscures the effects of hazard, but has nevertheless allowed us to identify new facets of value and to articulate them. We conclude this paper by suggesting several future research extensions. First, the addition of stochastic hypotheses would eventually lead to the emergence of new relevant and useful results regarding the drivers of value creation. Then, our theory provides new and powerful tools for further research, especially in the investigation of the accounting data published by companies, as this theory provides the means to infer companies' accounting schemes and would thus better explain the origin of their financial performance. We leave these interesting questions for future research.

References

Arya, A., Fellingham, J., Schroeder, D., & Young, R.(1996). Double entry bookkeeping and error correction. Ohio State University. https://cpb-us-w2.wpmucdn.com/u.osu.edu/dist/7/36891/files/2019/06/DoubleEntry.pdf

Asness, C. S., Moskowitz, T. J., & Pedersen, L. H. (2013). Value and momentum everywhere. *The Journal of Finance*, 68(3), 929-985. https://doi.org/10.1111/jofi.12021

Ball, R., Gerakos, J., Linnainmaa, J. T., & Nikolaev, V. (2016). Accruals, cash flows, and operating profitability in the cross section of stock returns. *Journal of Financial Economics*, 121(1), 28-45. https://doi.org/10.1016/j.jfineco.2016.03.002

Balzer, W., & Mattessich, R. (1991). An axiomatic basis of accounting: A structuralist reconstruction. *Theory and Decision*, 30(3), 213-243. https://doi.org/10.1007/BF00132445

Barnes, P. (1987). The analysis and use of financial ratios: A review article. *Journal of Business Finance & Accounting*, 14(4), 449-461. https://doi.org/10.1111/j.1468-5957.1987.tb00106.x

Bernanke, B. S., & Kuttner, K. N. (2005). What explains the stock market's reaction to Federal Reserve policy? *The Journal of Finance*, 60(3), 1221-1257. https://doi.org/10.1111/j.1540-6261.2005.00760.x

Bhanja, N., Dar, A. B., & Tiwari, A. K. (2012). Are stock prices hedge against inflation? A revisit over time and frequencies in India. *Central European Journal of Economic Modelling and Econometrics*, 4(3), 199-213.

Bodie, Z. (1976). Common stocks as a hedge against inflation. *The journal of finance*, 31(2), 459-470. https://doi.org/10.1111/j.1540-6261.1976.tb01899.x

Boudoukh, J., & Richardson, M. (1993). Stock returns and inflation: A long-horizon perspective. The American economic review, 83(5), 1346-1355. https://www.jstor.org/stable/2117566

Boudoukh, J., Richardson, M., & Whitelaw, R. F. (1994). Industry returns and the Fisher effect. the Journal of Finance, 49(5), 1595-1615. https://doi.org/10.1111/j.1540-6261.1994.tb04774.x

Bracewell, R. (1965). *Convolution in the Fourier Transform and its Applications*. McGraw-Hill, New York.

Brahmasrene, T., Strupeck, C. D., & Whitten, D. (2004). Examining preferences in cash flow statement format. *CPA Journal*, 74(10), 58-60.

Butterworth, J. E. (1972). The accounting system as an information function. *Journal of Accounting Research*, 10(1), 1-27. https://doi.org/10.2307/2490216

Campbell, J. Y., & Vuolteenaho, T. (2004). Inflation illusion and stock prices. *American Economic Review*, 94(2), 19-23. https://www.aeaweb.org/articles?id=10.1257/0002828041301533

Cardao-Pito, T., & Silva Ferreira, J. (2018). 'Fair Value' accounting as the normative Fisherian phase of accounting. *Accounting History Review*, 28(3), 149-179. https://doi.org/10.1080/21552851.2018.1541000

Carlson, M. L., & Lamb, J. W. (1981). Constructing a theory of accounting - An axiomatic approach. *Accounting Review*, 56, 554-573. https://www.jstor.org/stable/246915

Chen, L., Da, Z., & Zhao, X. (2013). What drives stock price movements?. *The Review of Financial Studies,* 26(4), 841-876. https://doi.org/10.1093/rfs/hht005

Cheung, W. M., & Jiang, L. (2016). Does free cash flow problem contribute to excess stock return synchronicity?. *Review of Quantitative Finance and Accounting*, 46(1), 123-140. https://doi.org/10.1007/s11156-014-0464-2

Churchill, N. C., & Mullins, J. W. (2001). How fast can your company afford to grow?. *Harvard Business Review*, 79(5), 135-4. https://hbr.org/2001/05/how-fast-can-your-company-afford-to-grow

Cohen, R. B., Polk, C., & Vuolteenaho, T. (2005). Money illusion in the stock market: The Modigliani-Cohn hypothesis. *The Quarterly Journal of Economics*, 120(2), 639-668. https://doi.org/10.1093/qje/120.2.639

Cooke, T., & Tippett, M. (2000). Double entry bookkeeping, structural dynamics and the value of the firm. *The British Accounting Review*, 32(3), 261-288. https://doi.org/10.1006/bare.2000.0135

Cooper, M. J., Gulen, H., & Schill, M. J. (2008). Asset growth and the cross-section of stock returns. *The Journal of Finance*, 63(4), 1609-1651. https://doi.org/10.1111/j.1540-6261.2008.01370.x

Corcoran, A. W. (1968). *Mathematical Applications in Accounting*. Harcourt, Brace & World, New York.

Da, Z. (2009). Cash flow, consumption risk, and the cross-section of stock returns. The Journal of Finance, 64(2), 923-956. https://doi.org/10.1111/j.1540-6261.2009.01453.x

Damodaran, A. (2007). Valuation Approaches and Metrics: A Survey of the Theory and Evidence, *Foundations and Trends® in Finance,* 1(8), 693-784. http://dx.doi.org/10.1561/0500000013

Damodaran, A. (2010). *Applied corporate finance.* John Wiley & Sons, Hoboken, NJ.

Deakin, E. B. (1976). Distributions of financial accounting ratios: some empirical evidence. *The Accounting Review*, 51(1), 90-96. https://www.jstor.org/stable/245375

Dreyer, J., Erasmus, P., Morrison, J., & Hamman, W. (2013). Sustainable company growth as measured by cash flow. *Management Dynamics: Journal of the Southern African Institute for Management Scientists,* 22(2), 16-28.

Durai, S. R. S., & Bhaduri, S. N. (2009). Stock prices, inflation and output: Evidence from wavelet analysis. *Economic Modelling*, 26(5), 1089-1092.

Ellerman, D. P. (1985). The mathematics of double entry bookkeeping. *Mathematics Magazine,* 58(4), 226-233. https://doi.org/10.1080/0025570X.1985.11977191

Fama, E. F. (1981). Stock returns, real activity, inflation, and money. *The American Economic Review,* 71(4), 545-565. https://www.jstor.org/stable/1806180

Fama, E. F., & French, K. R. (2006). Profitability, investment and average returns. *Journal of financial economics*, 82(3), 491-518. https://doi.org/10.1016/j.jfineco.2005.09.009

Fama, E. F., & French, K. R., (2008). Dissecting anomalies. *The Journal of Finance*, 63(4), 1653-1678. https://doi.org/10.1111/j.1540-6261.2008.01371.x

Fama, E. F., & Schwert, G. W. (1977). Asset returns and inflation. *Journal of Financial Economics,* 5(2), 115-146. https://doi.org/10.1016/0304-405X(77)90014-9

Foerster, S., Tsagarelis, J., & Wang, G. (2017). Are cash flows better stock return predictors than profits?. *Financial Analysts Journal,* 73(1), 73-99. https://doi.org/10.2469/faj.v73.n1.2

Frecka, T. J., & Hopwood, W. S. (1983). The effects of outliers on the cross-sectional distributional properties of financial ratios. *Accounting Review*, January, 115-128. https://www.jstor.org/stable/246646

Gaffikin, M. J. (2003). The a priori wars: The modernisation of accounting thought. *Accounting Forum,* 27(3), 291-311. https://www.tandfonline.com/doi/abs/10.1111/1467-6303.00107

Gaffikin, M. (2005). Creating a Science of Accounting: accounting theory to 1970. UOW, School of Accounting & Finance, Working Paper 05/08. https://ro.uow.edu.au/accfinwp/49/

Galí, J., & Gambetti, L. (2015). The effects of monetary policy on stock market bubbles: Some evidence. *American Economic Journal: Macroeconomics*, 7(1), 233-57. DOI: 10.1257/mac.20140003

Gang, J., Qian, Z., & Xu, T. (2019). Investment horizons, cash flow news, and the profitability of momentum and reversal strategies in the Chinese stock market. *Economic Modelling*, 83, 364-371. https://doi.org/10.1016/j.econmod.2019.08.021

Gentili, L., & Giacomello, B. (2017). A new mathematical framework for the balance sheet dynamic modeling. University of Verona, Department of Economics: Working paper, 5. http://dse.univr.it/home/workingpapers/wp2017n5.pdf

Girardi, D., Giacomello, B., & Gentili, L. (2011). Budgeting Models and System Simulation: a dynamic approach. http://dx.doi.org/10.2139/ssrn.1994453

Givoly, D., Hayn, C., & Lehavy, R. (2009). The quality of analysts' cash flow forecasts. *The Accounting Review*, 84(6), 1877-1911. https://doi.org/10.2308/accr.2009.84.6.1877

Gray, R. M. (2006). Toeplitz and circulant matrices: A review. *Foundations and Trends® in Communications and Information Theory* 2(3), 155-239. http://dx.doi.org/10.1561/0100000006

Griffin, J. M., Ji, X., & Martin, J. S. (2003). Momentum investing and business cycle risk: Evidence from pole to pole. *The Journal of finance*, 58(6), 2515-2547. https://doi.org/10.1046/j.1540-6261.2003.00614.x

Hamman, W. D. (1996). Sustainable growth: A cash flow model-Investment Basics XXXIII. *Investment Analysts Journal,* 25(43), 57-61. https://doi.org/10.1080/10293523.1996.11082362

Hou, K., Karolyi, G. A., & Kho, B. C. (2011). What factors drive global stock returns?. *The Review of Financial Studies*, 24(8), 2527-2574. https://doi.org/10.1093/rfs/hhr013

Ijiri, Y. (1965). Axioms and structures of conventional accounting measurement. *The Accounting Review,* 40(1), 36-53. https://www.jstor.org/stable/242624

Ijiri, Y. (1967). *The foundations of accounting measurement: A mathematical, economic, and behavioral inquiry*. Prentice-Hall, Englewood Cliffs, N.J.

Ijiri, Y. (1971). Axioms for historical cost valuation: A reply. *Journal of Accounting Research,* 9(1), 181-187. https://doi.org/10.2307/2490213

Ijiri, Y. (1975). *Theory of accounting measurement*. Studies in Accounting Research. American Accounting Association, Sarasota, FL.

Ioannidis, C., Peel, D. A., & Peel, M. J. (2003). The time series properties of financial ratios: Lev revisited. *Journal of Business Finance & Accounting,* 30(5-6), 699-714. https://doi.org/10.1111/1468-5957.05201

Jansen, B. A. (2021). Cash flow growth and stock returns. *Journal of Financial Research,* 44(2), 371-402. https://doi.org/10.1111/jfir.12244

Jegadeesh, N., & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance*, 48(1), 65-91. https://doi.org/10.1111/j.1540-6261.1993.tb04702.x

Jensen, M. C. (1986). Agency costs of free cash flow, corporate finance, and takeovers. *The American Economic Review*, 76(2), 323-329. https://www.jstor.org/stable/1818789

Kaplan, S. N., & Ruback, R. S. (1995). The valuation of cash flow forecasts: An empirical analysis. *The journal of Finance*, *50*(4), 1059-1093. https://doi.org/10.1111/j.1540-6261.1995.tb04050.x

Kaplan, S. N., & Ruback, R. S. (1996). The market pricing of cash flow forecasts: Discounted cash flow vs. the method of "comparables". *Journal of applied corporate finance*, *8*(4), 45-60. https://doi.org/10.1111/j.1745-6622.1996.tb00682.x

Karels, G. V., & Prakash, A. J. (1987). Multivariate normality and forecasting of business bankruptcy. *Journal of Business Finance & Accounting,* 14(4), 573-593. https://doi.org/10.1111/j.1468-5957.1987.tb00113.x

Kim, S., & In, F. (2005). The relationship between stock returns and inflation: new evidence from wavelet analysis. *Journal of empirical finance*, 12(3), 435-444. https://doi.org/10.1016/j.jempfin.2004.04.008

Krishnan, G. V., & Largay III, J. A. (2000). The predictive ability of direct method cash flow information. *Journal of Business Finance & Accounting,* 27(1-2), 215-245. https://doi.org/10.1111/1468-5957.00311

Larrain, B., & Yogo, M. (2008). Does firm value move too much to be justified by subsequent changes in cash flow?. *Journal of Financial Economics,* 87(1), 200-226. https://doi.org/10.1016/j.jfineco.2007.01.002

Levy, R. A. (1967). Relative strength as a criterion for investment selection. *The Journal of finance*, 22(4), 595-610. https://doi.org/10.2307/2326004

Leech, S. A. (1986). The theory and development of a matrix-based accounting system. *Accounting and Business Research,* 16(64), 327-341. https://doi.org/10.1080/00014788.1986.9729333

Lin, Q., & Lin, X. (2021). Cash conversion cycle and aggregate stock returns. *Journal of Financial Markets,* 52, 100560. https://doi.org/10.1016/j.finmar.2020.100560

Lintner, J. (1975). Inflation and security returns. *The journal of finance*, 30(2), 259-280. https://doi.org/10.2307/2978713

Mao, M. Q., & Wei, K. J. (2016). Cash-flow news and the investment effect in the cross section of stock returns. *Management Science*, 62(9), 2504-2519. https://doi.org/10.1287/mnsc.2015.2235

Markarian, G. (2018). The role of Irving Fisher in the development of fair value accounting thought. *Accounting History Review,* 28(3), 181-190. https://doi.org/10.1080/21552851.2018.1542230

Mattessich, R. (1957). Towards a general and axiomatic foundation of accountancy–with an introduction to the matrix formulation of accounting systems. *Accounting Research,* 8(4), 328-355.

Mattessich, R. (1958). Mathematical models in business accounting. *The Accounting Review,* 33(3), 472-481. https://www.jstor.org/stable/241190

Mattessich, R. (1964). *Accounting and Analytical Methods - Measurement and Projection of Income and Wealth in the Micro- and Macro-Economy*. Richard D. Irwin, Homewood, IL.

McLeay, S., & Stevenson, M. (2009). Modelling the longitudinal properties of financial ratios. *Applied Financial Economics*, 19(4), 305-318. https://doi.org/10.1080/09603100802167270

Mcleay, S., & Trigueiros, D. (2002). Proportionate growth and the theoretical foundations of financial ratios. *Abacus*, 38(3), 297-316. https://doi.org/10.1111/1467-6281.00111

Modigliani, F., & Cohn, R. A. (1979). Inflation, rational valuation and the market. *Financial Analysts Journal,* 35(2), 24-44. https://doi.org/10.2469/faj.v35.n2.24

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of Financial Economics*, 108(1), 1-28. https://doi.org/10.1016/j.jfineco.2013.01.003

Pae, J., & Yoon, S. S. (2012). Determinants of analysts' cash flow forecast accuracy. *Journal of Accounting, Auditing & Finance,* 27(1), 123-144. https://doi.org/10.1177%2F0148558X11409148

Peel, D. A., Peel, M. J., & Venetis, I. A. (2004). Further empirical analysis of the time series properties of financial ratios based on a panel data approach. *Applied Financial Economics,* 14(3), 155-163. https://doi.org/10.1080/0960310042000187342

Poularikas, A. D. (1999). *The Handbook of Formulas and Tables for Signal Processing*. CRC Press LLC, Boca Raton

Rappaport, A. (2005). The economics of short-term performance obsession. Financial Analysts Journal, 61(3), 65-79. https://doi.org/10.2469/faj.v61.n3.2729

Ritter, J. R., & Warr, R. S. (2002). The decline of inflation and the bull market of 1982–1999. *Journal of financial and quantitative analysis*, 37(1), 29-61. https://doi.org/10.2307/3594994

Ross, S. A., Westerfield, R. W., & Jordan B. D. (2015). *Fundamentals of corporate finance*. McGraw-Hill Professional, New York.

Schotman, P. C., & Schweitzer, M. (2000). Horizon sensitivity of the inflation hedge of stocks. Journal of empirical Finance, 7(3-4), 301-315. https://doi.org/10.1016/S0927-5398(00)00013-X

Sharpe, S. A. (2002). Reexamining stock valuation and inflation: The implications of analysts' earnings forecasts. *Review of Economics and Statistics*, 84(4), 632-648. https://doi.org/10.1162/003465302760556468

Solnik, B., & Solnik, V. (1997). A multi-country test of the Fisher model for stock returns. *Journal of International Financial Markets, Institutions and Money*, 7(4), 289-301. https://doi.org/10.1016/S1042-4431(97)00024-3

Sudarsanam, P. S., & Taffler, R. J. (1995). Financial ratio proportionality and inter-temporal stability: An empirical analysis. *Journal of Banking & Finance*, 19(1), 45-60. https://doi.org/10.1016/0378-4266(94)00044-4

Tippett, M. (1978). The axioms of accounting measurement. *Accounting and Business Research,* 8(32), 266-278. https://doi.org/10.1080/00014788.1978.9728729

Tippett, M. (1990). An induced theory of financial ratios. *Accounting and Business Research,* 21(81), 77-85. https://doi.org/10.1080/00014788.1990.9729406

Tippett, M., & Whittington, G. (1995). An empirical evaluation of an induced theory of financial ratios. *Accounting and Business Research,* 25(99), 208-218. https://doi.org/10.1080/00014788.1995.9729943

Titman, S., Wei, K. J., & Xie, F. (2004). Capital investments and stock returns. *Journal of financial and Quantitative Analysis*, 39(4), 677-700. https://doi.org/10.1017/S0022109000003173

Tiwari, A. K., Dar, A. B., Bhanja, N., Arouri, M., & Teulon, F. (2015). Stock returns and inflation in Pakistan. *Economic Modelling*, 47, 23-31. https://doi.org/10.1016/j.econmod.2014.12.043

Van Riel, R. V., & Van Gulick, R. (2019). *Scientific Reduction*. Stanford Encyclopedia of Philosophy, Stanford. https://plato.stanford.edu/archives/spr2019/entries/scientific-reduction/

Vernimmen, P., Quiry, P., Dallocchio, M., Le Fur, Y., & Salvi, A. (2017) *Corporate finance: theory and practice*. John Wiley & Sons, Hoboken, NJ.

Vuolteenaho, T. (2002). What drives firm-level stock returns?. *The Journal of Finance*, *57*(1), 233-264. https://doi.org/10.1111/1540-6261.00421

Wang, B. (2019). The cash conversion cycle spread. *Journal of Financial Economics,* 133(2), 472-497. https://doi.org/10.1016/j.jfineco.2019.02.008

Watanabe, A., Xu, Y., Yao, T., & Yu, T. (2013). The asset growth effect: Insights from international equity markets. *Journal of Financial Economics*, 108(2), 529-563. https://doi.org/10.1016/j.jfineco.2012.12.002

Wen, Q. (2019). Asset growth and stock market returns: A time-series analysis. *Review of Finance,* 23(3), 599-628. https://doi.org/10.1093/rof/rfy018

Whittington, G. (1980). Some basic properties of accounting ratios. *Journal of Business Finance & Accounting*, 7(2), 219-232. https://doi.org/10.1111/j.1468-5957.1980.tb00738.x

Whittington, G., & Tippett, M. (1999). The Components of Accounting Ratios as Co-integrated Variables. *Journal of Business Finance & Accounting,* 26(9-10), 1245-1273. https://doi.org/10.1111/1468-5957.00296

Williams, T. H., & Griffin, C. H. (1964). Matrix theory and cost allocation. *The Accounting Review*, 39(3), 671-678. https://www.jstor.org/stable/242462

Zeidan, R., & Shapir, O. M. (2017). Cash conversion cycle and value-enhancing operations: Theory and evidence for a free lunch. *Journal of Corporate Finance*, 45(C), 203-219. https://doi.org/10.1016/j.jcorpfin.2017.04.014

Appendix A

A.1 Mathematical formulations of accounting

Our theory describes a contingent economic activity in plain accounting terms a series (noted "A") of identical and contingent *economic operations* (a_n), achieved per period (n). This activity forms a vector noted $|a_1, a_2, \dots, a_{t-1}, a_t|$. This activity is described using the notion of *scheme* (Section 3.2) providing a complete accounting description of a single *economic operation*. A specific scheme is noted "S" and represents of a *group of vectors* - one vector for each account required to describe the economic operation. Examples are provided in Section 3.2. Therefore, a scheme (S) describes each of the economic operations making up the notion of economic activity (A).

It remains to combine S and A, so as to describe economic activities in the form of a balance sheet, a group of vectors representing the evolution of the balance of accounts over the periods. Though, combining S with A implies combining each of the account vectors that constitute S with A through the same procedure. So we propose to keep our focus on the combination of a single account vector (cash position for example), noted $|s_1,\ s_2,\ \dots, s_{t-1},\ s_t|$, with its activity A, noted $|a_1,\ a_2,\ \dots,\ a_{t-1},\ a_t|$. We propose to combine these two vectors, first of all in a visual and intuitive way to give a sense to the calculations, before establishing in a second time their correspondence with standard mathematical operations, but certainly less evocative. Hence, the vector $|s_1,\ s_2,\ \dots,\ s_{t-1},\ s_t|$ is used to form a matrix (noted "M" in Equation A1 below) - each row corresponding to a specific period. The components of each new row-period are shifted one column to the right to reflect the flow of time relative to the previous row-period. We associate to each row of M the number of economic operations performed during the corresponding period (components of "A"). The product of M and A, noted "M*A" in Equation A1 below, corresponds to a new matrix whose rows are obtained by multiplying each row-period of M with the number of operations corresponding of "A" :

$$\begin{bmatrix} s_1 & s_2 & \cdots & s_{t-1} & s_t \\ 0 & s_1 & \cdots & s_{t-2} & s_{t-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & s_1 & s_2 \\ 0 & 0 & \cdots & 0 & s_1 \end{bmatrix} * \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_{t-1} \\ a_t \end{bmatrix} = \begin{bmatrix} s_1.a_1 & s_2.a_1 & \cdots & s_{t-1}.a_1 & s_t.a_1 \\ 0 & s_1.a_2 & \cdots & s_{t-2}.a_2 & s_{t-1}.a_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & s_1.a_{t-1} & s_2.a_{t-1} \\ 0 & 0 & \cdots & 0 & s_1.a_t \end{bmatrix} \tag{A1}$$

$$M \qquad\qquad \mathrm{A} \qquad\qquad M * \mathrm{A}$$

These premises allow us to describe the evolution of cash positions in the form of a vector named "C", by calculating the sums of the different columns of "M*A":

$$C = \left| \sum_{i=1}^{1} s_i.a_{2-i}, \quad \sum_{i=1}^{2} s_i.a_{3-i}, \quad \ldots, \quad \sum_{i=1}^{t-1} s_i.a_{t-i}, \quad \sum_{i=1}^{t} s_i.a_{t+1-i} \right| \tag{A2}$$

Again, the calculation of the evolution of the balances of the other accounts follows the same procedure. These prerequisites have enabled us to present the calculation of accounting balances in a visual and intuitive form and we will now establish their correspondence with more standard mathematical operations. This correspondence comes simply by noticing that the components of C (Equation A2) can be calculated by means of a standard matrix product realized with the same objects (M and A) :

$$\mathrm{C} = \begin{vmatrix} s_1 & 0 & \cdots & 0 & 0 \\ s_2 & s_1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ s_{t-1} & s_{t-2} & \cdots & s_1 & 0 \\ s_t & s_{t-1} & \cdots & s_2 & s_1 \end{vmatrix} . \begin{vmatrix} a_1 \\ a_2 \\ \cdots \\ a_{t-1} \\ a_t \end{vmatrix} = M^T.\mathrm{A} \tag{A3}$$

M^T stands for the transpose matrix of M, which we multiply with A by means of a classical matrix product. We emphasize that the constant diagonals, from left to right, of the matrix M are characteristic of a "Toeplitz" matrix, whose transposed version (M^T) is also a Toeplitz matrix (e.g., Poularikas 1999). This point establishes the equivalence between our calculation the balance of the accounts (Equations A2 and A3) and a "discrete convolution product" (DCP, e.g., Gray 2006) performed between S and A.

We note these two possibilities of calculation (matrix product and DCP) under a common "bracket" notation:

$$\mathrm{C} = \langle \mathrm{S} | \mathrm{A} \rangle \tag{A4}$$

Therefore, repeating the calculation on all the other accounts of the scheme (for example, receivables, VAT, turnover) makes it possible to calculate the balance sheet related to the economic activity. We illustrate these calculations concretely on the basis of a fictitious economic activity, consisting of purchasing non-performing loan portfolios and collecting on these portfolios. Table 1 (Section 3.2) illustrates the scheme "S" associated with the acquisition of a "standard" portfolio and the collection of cash in the months following its acquisition. To this scheme, which describes the purchase of a "standard" portfolio, we associate an economic activity "A" describing the number of non-performing loan portfolios purchased each month during a twelve month sequence :

A = |1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3|.

Table 4 details the matrix form of the calculation of the evolution of the cash position (Equation A1)

Table 4. Calculation of cash positions through a combination of M and A (M*A) :

"M" matrix - based on "S" cash position vector (in grey)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**		**"A" Vector Activity**
-100	-75	-39	-11	8	18	20	20	20	20	20	20		1
0	-100	-75	-39	-11	8	18	20	20	20	20	20		2
0	0	-100	-75	-39	-11	8	18	20	20	20	20		3
0	0	0	-100	-75	-39	-11	8	18	20	20	20		1
0	0	0	0	-100	-75	-39	-11	8	18	20	20		2
0	0	0	0	0	-100	-75	-39	-11	8	18	20	*	3
0	0	0	0	0	0	-100	-75	-39	-11	8	18		1
0	0	0	0	0	0	0	-100	-75	-39	-11	8		2
0	0	0	0	0	0	0	0	-100	-75	-39	-11		3
0	0	0	0	0	0	0	0	0	-100	-75	-39		1
0	0	0	0	0	0	0	0	0	0	-100	-75		2
0	0	0	0	0	0	0	0	0	0	0	-100		3

"M*A" Matrix

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
-100	-75	-39	-11	8	18	20	20	20	20	20	20
0	-200	-150	-78	-22	16	36	40	40	40	40	40
0	0	-300	-225	-117	-33	24	54	60	60	60	60
0	0	0	-100	-75	-39	-11	8	18	20	20	20
0	0	0	0	-200	-150	-78	-22	16	36	40	40
0	0	0	0	0	-300	-225	-117	-33	24	54	60
0	0	0	0	0	0	-100	-75	-39	-11	8	18
0	0	0	0	0	0	0	-200	-150	-78	-22	16
0	0	0	0	0	0	0	0	-300	-225	-117	-33
0	0	0	0	0	0	0	0	0	-100	-75	-39
0	0	0	0	0	0	0	0	0	0	-200	-150
0	0	0	0	0	0	0	0	0	0	0	-300

"C" vector - Evolution of cash position

-100	**-275**	**-489**	**-414**	**-406**	**-488**	**-334**	**-292**	**-368**	**-214**	**-172**	**-248**

Table 5 presents the calculation of the complete balance sheet related to this activity and its evolution over the twelve month period.

Table 5. Balance sheet related to a loan portfolio activity (cash position highlighted)

Ass./Liab.	Accounts	1	2	3	4	5	6	7	8	9	10	11	12
Assets	Asset value	100	282	517	479	506	625	516	512	625	516	512	625
Assets	**Cash**	**-100**	**-275**	**-489**	**-414**	**-406**	**-488**	**-334**	**-292**	**-368**	**-214**	**-172**	**-248**
Total Assets		**0**	**7**	**28**	**65**	**100**	**137**	**182**	**220**	**257**	**302**	**340**	**377**
Equity	Collection (earnings)	0	30	130	330	560	800	1082	1346	1592	1874	2138	2384
Equity	Coll. costs (earnings)	0	-5	-19	-44	-66	-88	-116	-138	-160	-188	-210	-232
Equity	Amort. (earnings)	0	-18	-83	-221	-394	-575	-784	-988	-1175	-1384	-1588	-1775
Total Equity + Liabilities		**0**	**7**	**28**	**65**	**100**	**137**	**182**	**220**	**257**	**302**	**340**	**377**

A.2. Reduction of the FV to our mathematical theory of accounting

We start from the current formulation of the FV:

$$FV = \sum_{i=1}^{t} CF_i.(1+r)^{t-i} \tag{A5}$$

We lighten notation, setting $(1+r) = \alpha$ and $CF_t = \Phi_t$ so that:

$$FV = \sum_{i=1}^{t} \Phi_i.\alpha^{t-i} \tag{A6}$$

This light notation makes it easier to develop each component of the sum $\Phi_i.\alpha^{t-i}$ over time:

$$\left.\begin{array}{lcl} \Phi_1.\alpha^{t-1} & = & \Phi_1.\alpha^0 + \Phi_1.(\alpha^1-\alpha^0) + \cdots + \Phi_1.(\alpha^{t-2}-\alpha^{t-3}) + \Phi_1.(\alpha^{t-1}-\alpha^{t-2}) \\ \Phi_2.\alpha^{t-2} & = & \Phi_2.\alpha^0 + \Phi_2.(\alpha^1-\alpha^0) + \ldots + \Phi_2.(\alpha^{t-2}-\alpha^{t-3}) \\ \cdots & \cdots & \cdots \\ \Phi_{t-1}.\alpha^1 & = & \Phi_{t-1}.\alpha^0 + \Phi_{t-1}.(\alpha^1-\alpha^0) \\ \Phi_t.\alpha^0 & = & \Phi_t.\alpha^0 \end{array}\right\} \tag{A7}$$

This development of each component of the FV provides new components that are displayed in form of a matrix, in which the components' sum remains equal to the FV:

$$\begin{bmatrix} \Phi_1.\alpha^0 & \Phi_1.(\alpha^1-\alpha^0) & \cdots & \Phi_1.(\alpha^{t-2}-\alpha^{t-3}) & \Phi_1.(\alpha^{t-1}-\alpha^{t-2}) \\ \Phi_2.\alpha^0 & \Phi_2.(\alpha^1-\alpha^0) & \cdots & \Phi_2.(\alpha^{t-2}-\alpha^{t-3}) & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \Phi_{t-1}.\alpha^0 & \Phi_{t-1}.(\alpha^1-\alpha^0) & \cdots & 0 & 0 \\ \Phi_t.\alpha^0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{A8}$$

Next, we display the matrix lines in diagonals:

$$\begin{bmatrix} \Phi_1.\alpha^0 & \Phi_2.\alpha^0 & \cdots & \Phi_{t-1}.\alpha^0 & \Phi_t.\alpha^0 \\ 0 & \Phi_1.(\alpha^1-\alpha^0) & \cdots & \Phi_{t-2}.(\alpha^1-\alpha^0) & \Phi_{t-1}.(\alpha^1-\alpha^0) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \Phi_1.(\alpha^{t-2}-\alpha^{t-3}) & \Phi_2.(\alpha^{t-2}-\alpha^{t-3}) \\ 0 & 0 & \cdots & 0 & \Phi_1.(\alpha^{t-1}-\alpha^{t-2}) \end{bmatrix} \tag{A9}$$

We recognize the pattern of a DCP (Equation A1) performed between the CF vector $|\Phi_1, \Phi_1, \dots, \Phi_{t-1}, \Phi_t|$, equal to $\langle \Delta S|A\rangle$, and the marginal capitalization vector ΔF, equal to $|\alpha^0, \alpha^1-\alpha^0, \dots, \alpha^{t-2}-\alpha^{t-3}, \alpha^{t-1}-\alpha^{t-2}|$. The sum of the vector generated by the DCP is equal to the FV, which is reformulated as following:

$$\mathrm{FV} = \sum\big\langle \langle \Delta S|A\rangle \big| \Delta F\big\rangle \tag{A10}$$

The switching derivation property of DCP (Bracewell, 1965) leads to a compact FV' formulation:

$$\mathrm{FV} = \sum\langle \Delta_2 S|A|F\rangle \tag{A11}$$

A.3. Formulation of the CFSGR

We formulate the CFSGR as the growth rate that cancels the CF generated in each period. That is, we seek a growth rate of economic activity "A," such that the components of the vector of CF generated by Equation (6) ($\langle \Delta S|A\rangle$) are zero. We first define A with an initial level of activity (economic operations) equal to Q, assumed to grow at a constant rate "g" in each period. Therefore, A is equal to $|Q.(1+g)^1, Q.(1+g)^2, \dots, Q.(1+g)^{t-1}, Q.(1+g)^t|$, whose formulation we lighten by posing $\gamma = 1+g$, which simplifies its form to $Q.|\gamma^1, \gamma^2, \dots, \gamma^{t-1}, \gamma^t|$. We then define the vector ΔS equal to the variations of the components of the cash balance vector of S: $|s_1, s_2 - s_1, \dots, s_{t-1} - s_{t-2}, s_t - s_{t-1}|$, in which we simplify the notation by $|\Delta s_1, \Delta s_2, \dots, \Delta s_{t-1}, \Delta s_t|$. We now compute the CF, combining ΔS and A, through our matrix formulation of Equation (A1), which provides a visual form to our calculations.

$$Q.\begin{bmatrix} \Delta s_1.\gamma^1 & \Delta s_2.\gamma^1 & \cdots & \Delta s_{t-1}.\gamma^1 & \Delta s_t.\gamma^1 \\ 0 & \Delta s_1.\gamma^2 & \cdots & \Delta s_{t-2}.\gamma^2 & \Delta s_{t-1}.\gamma^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \Delta s_1.\gamma^{t-1} & \Delta s_2.\gamma^{t-1} \\ 0 & 0 & \cdots & 0 & \Delta s_1.\gamma^t \end{bmatrix} \tag{A12}$$

As seen in Appendix A.1, this matrix formulation allows us to calculate the CF of each period by summing the corresponding column. Before we get to the analytical formulation of the CFSGR, we simply point out that the accounts of a scheme are supposed to stabilize after a certain number of periods, in other words, any economic operation is supposed to stop generating accounting entries after a certain time. Therefore, we consider that the components of ΔS are zero beyond the period “X.” These preliminaries lead us to formulate the CFSGR by stipulating that the sum of each of the columns must be zero (if the columns contain all the “X” components of ΔS). As such, we arbitrarily take the last column of the matrix (A12):

$$Q.\gamma^t.\sum_{i=1}^{X}\Delta s_i.\gamma^{-i} = 0 \tag{A13}$$

The factor $Q.\gamma^t$ can be eliminated from our formulation. We return to our change of variable $\gamma = 1 + g$, so as to formulate the CFSGR as a solution of the variable “g”:

$$\sum_{i=1}^{X}\Delta s_i.(1+g)^{-i} = 0 \tag{A14}$$

This equation has as many solutions as the ΔS has sign changes, according to the “Descartes rule”. Given the usual growth rates of companies, we only consider the closest solution to zero, noted “λ”.

A.4. Equilibrium investment horizon

Here we measure up to which investment horizon the erosive effects of growth, financing and inflation are affecting CF, the FV (Section 5.2) and DCF (Section 5.3). Beyond this horizon, the erosive effects give way to positive effects from the composition of positive rates of growth, financing and inflation over time. This phenomenon can be seen in the transition from a 12-month horizon figures to 24-month in Figures 1 and 2 (Sections 5.2 and 5.3). These observations suggest that this equilibrium investment horizon (noted H) is dependent on the fundamental determinants of CF and the passing of time itself. Therefore, we begin by measuring H in terms of the behavior of CF, whose links to the FV and DCF will allow us to generalize our results. So, we measure H by identifying the point at which the effects of growth composition allow CF (Equation A13) to catch up with the erosive effects and reach the same level that CF would have reached without growth, that is, the unit margin (M) multiplied by the starting activity (Q) :

$$Q.(1+g)^H.\sum_{i=1}^{X}\Delta s_i.(1+g)^{-i} = M.Q \tag{A15}$$

This relation provides a direct formula for H :

$$H = X + \log_{1+g}\frac{M}{\sum_{i=1}^{X}\Delta s_i.(1+g)^{X-i}} \tag{A16}$$

In other words, the equilibrium horizon H is equal to the addition of the length of the activity scheme (X), with the time necessary for positive long-term composition effects to start catching up with short-term erosive effects, equal to $\log_{1+g} \frac{M}{\sum_{i=1}^{X} \Delta s_i.(1+g)^{X-i}}$. Resting on CF alone, this formulation of H can be generalized simply to the FV by noting that the effects of financing and inflation are symmetric to those of growth. Therefore, H corresponds to a common horizon for them. The discounting of the FV, leading to the DCF, naturally breaks the symmetry between the effects of growth and inflation on one side and those of financing on the other. H is therefore only modified in DCF with respect to the effects of financing, which naturally move this horizon away to infinity. An example, that of a loan portfolio (Table 1 in Section 3.2), gives a numerical evaluation of H at 16.96 months, for a growth rate equal to 0.01%. This positive but minimal growth rate allows us to measure H at the earliest stage of its appearance - indeed and for a growth rate of 1%, H reaches 18.06 months. Equation (A16) is not sufficient to get a simple and general idea of the level of H, given the infinite possibilities for the unit margin M and the shape of the scheme. Equation (13) simplifies considerably the formulation of CF to the horizon H, noted CF_H :

$$CF_H \approx M.\left(1 - \frac{g}{\lambda}\right).(1+g)^H.Q \tag{A17}$$

We look for the level of H by positing again the equality between CF_H and $M.Q$:

$$M.\left(1 - \frac{g}{\lambda}\right).(1+g)^H.Q \approx M.Q \tag{A18}$$

We finally deduce a condensed formulation of H :

$$H \approx \log_{1+g} \frac{\lambda}{\lambda - g} \tag{A19}$$

This approximates H as function of λ and growth catching most of the complex interplay of prices and payment terms. To take our example, that of the loan portfolio scheme (Table 1 in Section 3.2), H ≈ 13.78, to compare to the previous exact value equal at 16.96 (-18,73%), as a proxy. Table 6 illustrates the approximate values of H as a function of λ as the only variable. This shows that low levels of λ push H away and allow the erosive effects to occur, while high levels of λ tend to pull H closer in time, allowing to occur earlier the positive effects of growth composition on CF.

Table 6. Approximation of H (in months) as a function of λ, ranging from 1% to 15%, and for a 0.01% growth :

λ	H
1%	100.51
2%	50.13
3%	33.39
4%	25.03
5%	20.02
6%	16.68
7%	14.30
8%	12.51
9%	11.12
10%	10.01
11%	9.10
12%	8.34
13%	7.70
14%	7.15
15%	6.67

This study is natural for positive values of λ, since the IRR is one of its facets (Section 5.1) and it is expected that the IRR should be as high as possible, both in absolute terms and in relation to financing in the standard DCF model. The measure of H with a minimal rate of growth (0.01%) responds to the need to capture the moment when the effects of growth composition finally just start to take effect, for a positive λ.

H marks the beginning of a process yielding its full effects with time. H also provides a consistent explanation for the erosive effects of growth, financing, and inflation, visible on stock returns and discussed in Section 6.2. As a first remark, is that it is natural to assume that most companies have low levels of profitability and therefore IRR, rather than the opposite. The IRR being a facet of λ (Section 5.2), most companies should therefore have low levels of λ. Table 6 shows that H should be relatively high for most companies, i.e. higher than 12 months, especially if one takes into account the linearization gap. The second remark is that the investment horizon in financial markets, noted $\widetilde{H}$, is generally measured in studies at levels less than or equal to 12 months (Section 6.1). $\widetilde{H}$ being inferior to H for most of companies, this would naturally explain the erosive effects of growth, financing and inflation on stock returns (Section 6.2).