A Simple No-Arbitrage Approach to Pricing Single-Name Credit Risky Securities

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Abstract

This study introduces a simple approach to pricing single-name credit risky securities. The method relies on a set of no-arbitrage conditions that must be verified by three well-established building blocks in credit risk pricing, which relate their values for any two consecutive maturities and possible defaulting periods. It is shown that, together with the term structure of credit default swap spreads, the reported conditions allow for a straightforward estimation of the term structure of said building blocks that circumvents the estimation of risk-neutral default probabilities or hazard rates. The practical implementation of this method is illustrated with a case study.

JEL classification: G12, G13, G14

Keywords: Credit risk pricing, term structure of credit default swap spreads, spot and forward credit default swap contracts.

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1. Introduction

The aim of this study is to introduce a simple no-arbitrage approach to pricing single-name credit risky securities. Presently, the two possible approaches are structural and reduced-form models. While they are conceptually different, they both rely on modeling risk-neutral default probabilities, or hazard rates.

In structural models, default occurs the first time that the firm asset value crosses a critical lower bound or default barrier.[1] Structural models are theoretically appealing because they provide economic reasoning for the event of default based on fundamentals. Unfortunately, the latent nature of most of the variables and parameters involved (e.g., firm asset value and volatility) makes these models challenging to implement in practice. Moreover, even if we were to ignore this problem, replicating the observed market prices of credit risky securities may require a very elaborate model that incorporates, among other elements, stochastic firm asset volatility and jumps (Du, Elkamhi, and Ericsson, 2019).

In reduced-form models, default is governed by an exogenous hazard rate process that determines the default probability at each point in time, conditional on no-previous default. The observed market prices of credit risky securities are then used to calibrate this hazard rate process under the risk-neutral probability measure, in such a way that it can be finally applied in the pricing of other credit risky positions.[2] This sensible approach

[1] A non-exhaustive list of structural models would include the seminal work of Merton (1974), in addition to Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), and, more recently, Du, Elkamhi, and Ericsson (2019).

[2] Reduced-form models were initiated by Jarrow and Turnbull (1995) and followed, among others, by Jarrow, Lando and Turnbull (1997), Lando (1998), and Duffie and Singleton (1999). A detailed description of both structural and reduced-form models can be found in Duffie and Singleton (2003), Schönbucher (2003), Lando (2004), and O'Kane (2008). Madan and Unal (2000) represents an example of a hybrid (structural hazard rate) model.

has its costs. First, an exogenous hazard rate process needs to be assumed. Second, the price of each credit risky security must be derived as an *explicit* function of the corresponding risk-neutral hazard rate process. In practice, this does not need to be a simple task. Finally, because each security's price is a non-trivial function of the risk-neutral hazard rate process, its calibration based on actual market prices requires the implementation of root-search algorithms. Despite these challenges, reduced-form models represent the fastest and most accurate approach available at this moment to mark-to-market credit risky positions.

The method proposed in this study draws on three main elements. First, the price of most single-name credit risky securities can be expressed as a simple function of three well-established building blocks, or *credit risk discount factors* (CRDF), initially defined in Lando (1998). Among those securities are credit default swaps (CDS) contracts with different maturities. Second, in a discrete time economy, a set of no-arbitrage conditions can be derived between the value of those CRDF for any two consecutive maturities and possible defaulting times. The assumption of a discrete time economy is, thus, critical but not restrictive, as the time interval can be arbitrarily small. The sole condition is that it is strictly positive. Finally, if CDS spreads are available with maturities fitting all future time periods, then a system of equations exists that allows the bootstrap of such CRDF for all possible maturities. Practically, a sensible implementation of the method is achieved by assuming an economy with a daily time interval and that a term structure of CDS spreads (TSCDS) with corresponding maturities exists that fits the result of a linear interpolation between observable CDS spreads.

The main advantage of the proposed method—that avoids the estimation of risk-neutral default probabilities or hazard rates—lies precisely on its simplicity. As previously said, the prices of the most common single-name credit risky securities (e.g., bonds, CDS, and forward CDS) are simple functions of the CRDF and, as this study shows, the same simple structure applies for the no-arbitrage conditions between those CRDF. This implies that both the bootstrapping process and the posterior mark-to-market of other credit risky positions are particularly easy to implement. Specifically, the bootstrapping procedure is based on explicit closed-form solutions and does not involve any root-search algorithm.

A brief description of the most relevant applications of this method is provided. These include the mark-to-market of CDS positions, the pricing of risky bonds, and the pricing of forward CDS contracts. A less obvious but important application is the time decomposition of CDS spreads, discussed in more detail. The possible extension to portfolio management is also analyzed. Finally, the practical implementation of the method is illustrated with a case study: the time decomposition of sovereign CDS spreads during the Eurozone crisis.

The remainder of this paper is organized as follows. Section 2 defines the basic setting and introduces the no-arbitrage conditions between CRDF. Section 3 reviews the pricing of CDS contracts. Section 4 incorporates some additional assumptions and describes the bootstrapping process. Section 5 discusses some possible applications, including the time decomposition of CDS spreads. Section 6 presents the case study that illustrates the practical implementation of the method. A summary of the main conclusions is provided in Section 7.

2. Basic Setting and No-Arbitrage Conditions between Credit Risk Discount Factors

2.1. Setting

The focus of this study is the pricing of different credit risky assets and associated financial derivatives at current (non-defaulting) time 0. With this goal in mind, a simple discrete time economy with a daily time interval is assumed. Traded assets include (but are not necessarily restricted to) default-free and risky zero-coupon bonds of all possible maturities. These maturities are denoted T, and correspond to all future calendar dates up to time τ—that is, $\{\Delta, 2\Delta, \dots, \tau\}$, with $\Delta = 1/365$. The price of a default-free zero-coupon bond with nominal \$1 and maturity T is denoted $Z(T)$.[3] Regarding risky bonds, default may happen at any future calendar date and represents an absorbing state. The default time is denoted τ^d, while the minimum between τ^d and T is denoted L_d^T. In the event of default, bond holders receive (irrespective of the possible coupon) a fraction θ of its face value and the asset is liquidated. Markets are complete and arbitrage-free.

2.2. Credit Risk Discount Factors and No-Arbitrage Conditions

In our particular setting, the three basic CRDF are defined as follows:

- $A(T)$: The present value of an asset class A paying a constant annuity of \$$\Delta$ every Δ years until L_d^T (included).
- $B(T)$: The present value of an asset class B paying \$1 at τ^d, provided $\tau^d \leq T$.
- $C(T)$: The present value of an asset class C paying \$1 at T, provided $\tau^d > T$.

It is important to stress that, in the case of asset class A with maturity T, a default time $\tau^d \leq T$ implies the cancelation of the periodic stream of payments from $\tau^d + \Delta$

[3] Because all prices are determined at current time 0, simple notation is used that avoids emphasizing the present time 0.

onwards. This includes $\tau^d + \Delta$, but not τ^d itself. While such clarification is not necessary in a continuous time model (Lando, 1998), it will be a key element in our case. Also, the discrete time setting considered in this study allows to introduce a fourth convenient CRDF:

- $D(T)$: The present value of an asset class D paying \$1 at T, provided $\tau^d > T - \Delta$.

Hence, the difference between assets C and D with the same maturity T is that the payment of \$1 at T is conditional on survival at time T in the case of C, and on survival at the previous date $T - \Delta$ in the case of D.

Figure 1 depicts the structure of payments associated to the four contingent claims. Along with the assumptions made in section 2.1, this structure of payments implies two no-arbitrage conditions that must hold for any two consecutive maturities $T - \Delta$ and T.

<Figure 1 about here>

The first no-arbitrage condition (NAC1) relates $A(T)$, $A(T-\Delta)$ and $D(T)$:

$$A(T) = A(T-\Delta) + \Delta D(T), \tag{1}$$

with $A(0) = 0$.

Equation (1) reflects that the present value of a daily annuity of \$Δ paid until time T or default must be equal to the sum of: (a) the present value of a daily annuity of \$Δ paid until time $T - \Delta$ or default, and; (b) the present value of \$Δ paid with certainty at time T, conditional on no default at time $T - \Delta$ or before. This second component follows from the previous remark on the effect of a default event on the payments of an asset class A.

The second no-arbitrage condition (NAC2) that must hold for any two consecutive maturities $T-\Delta$ and T is as follows:

$$C(T)+B(T)-B(T-\Delta)=D(T), \tag{2}$$

with $B(0)=0$.

In the left-hand side of Equation (2), $C(T)$ is the present value of $1 paid at time T, conditional on no default at that time or before. Simultaneously, $B(T)-B(T-\Delta)$ equals the present value of $1 paid at T in the case of default at that precise moment, and not before. Taken as a whole, the left-hand side of Equation (2) equals the present value $1 paid with certainty at time T conditional on no default at time $T-\Delta$ or before, and this is exactly what $D(T)$ in the right-hand side of said equation represents. It is worth noting that the combination of Equations (1) and (2) leads to the following related condition:

$$A(T)=A(T-\Delta)+\Delta[C(T)+B(T)-B(T-\Delta)]. \tag{3}$$

Equation (3) provides a necessary relationship between the three core CRDF for any two consecutive maturities $T-\Delta$ and T. An important observation is that this equilibrium condition relies only on the structure of payments associated to assets A, B, and C and the assumptions made in section 2.1. In other words, it is not dependent on any particular risk-neutral pricing model.[4]

3. Pricing Credit Default Swaps with Credit Risk Discount Factors

The value of a position in a CDS contract with maturity T equals the difference between its premium leg and its protection leg. The daily structure of the premium leg is

[4] A further intuitive implication of Equation (3) is that $A(T)=\Delta\left[\sum_{h=1}^{T/\Delta} C(h\Delta)+B(T)\right]$.

described in Figure 2. This figure reflects a key feature of a CDS contract. Namely, while the annual premium per dollar of protected debt, cds, is usually paid in quarterly installments, the liquidation of the contract in the case of default implies the payment of the premium accrued since the last quarterly payment. Therefore, a non-defaulting state at a given day implies a consolidated right to accrue Δcds the following day, regardless of whether there is a default or not in that posterior day. If we further assume no counterparty risk coming from the side of the protection buyer, such consolidated right to accrue Δcds can be considered as a risk-free income at a given day, conditional on no default at the previous day. Because this structure of payments mimics that of asset A, scaled by cds, the present value of the premium leg is simply:

$$X(T) = cdsA(T), \tag{4}$$

where the nominal value of the protected bond is normalized to 1.

<Figure 2 about here>

The daily structure of the protection leg is shown in Figure 3. At any given day, the protection payment is 0 in the case of no default, and a fraction $(1-\theta)$ of the protected bond's face value in the case of default. Therefore, the payments' structure of the protection leg reproduces that of asset B scaled by $(1-\theta)$, and the same applies for its present value for a nominal of 1:

$$Y(T) = (1-\theta)B(T). \tag{5}$$

<Figure 3 about here>

The value of a long position in the CDS contract is, thus:

$$V(T) = (1-\theta)B(T) - cdsA(T), \quad (6)$$

and we obtain the break-even CDS spread, $cds(T)$, as the spread that satisfies $V(T) = 0$ (see also Duffie and Singleton, 2003):

$$cds(T) = \frac{(1-\theta)B(T)}{A(T)}. \quad (7)$$

4. Additional Assumptions and Bootstrapping of Credit Risk Discount Factors

All previous results are based on no-arbitrage arguments alone, which implies that they do not rely on any particular risk-neutral pricing model. Yet, a convenient additional assumption will be that the risk-free interest rate process and the default time τ^d are risk-neutrally independent (Jarrow and Turnbull, 1995; Jarrow, Lando and Turnbull, 1997; Duffie and Singleton, 2003). If we denote $S(T)$ the risk-neutral survival probability at time T (as seen at current time 0), this new assumption allows to decompose $C(T-\Delta)$ and $D(T)$ as follows: $C(T-\Delta) = Z(T-\Delta)S(T-\Delta)$; and $D(T) = Z(T)S(T-\Delta)$. If we further denote $f(T-\Delta, T) \equiv -(1/\Delta)log[Z(T)/Z(T-\Delta)]$ the forward risk-free rate between $T-\Delta$ and T, then we obtain:

$$D(T) = e^{-f(T-\Delta,T)\Delta}C(T-\Delta), \quad (8)$$

with $C(0) = 1$.

Let us now assume that $A(T-\Delta)$, $B(T-\Delta)$, and $C(T-\Delta)$ values are available for a given maturity $T-\Delta$. In such a case, and assuming that the forward rate $f(T-\Delta, T)$ is also available, Equations (1), (2), (7) and (8) lead to a system of three equations and three unknowns—$A(T)$, $B(T)$, and $C(T)$—with a simple closed-form solution:

$$A(T) = A(T - \Delta) + \Delta e^{-f(T-\Delta,T)\Delta} C(T - \Delta); \tag{9a}$$

$$B(T) = \frac{cds(T)A(T)}{(1 - \theta)}; \tag{9b}$$

$$C(T) = B(T - \Delta) - B(T) + e^{-f(T-\Delta,T)\Delta} C(T - \Delta). \tag{9c}$$

The solution to this system is in fact trivial and unique. Therefore, provided that a TSCDS is available that contains CDS spreads for all possible maturities $\{\Delta, 2\Delta, \dots, \tau\}$, it would be possible to bootstrap the corresponding term structure of CRDF based on Equation (9) and the initial values $A(0) = 0$, $B(0) = 0$, and $C(0) = 1$.[5] Unfortunately, the actual time interval between the maturity of observable CDS spreads ranges from six months to ten years, which clearly exceeds the required interval of one day. However, a sensible and practical solution can be obtained by linear interpolating the observed CDS spreads.

Table 1 presents a numerical example.[6] It assumes a TSCDS with standard maturities of 6m, 1y, 2y, 3y, 4y, 5y, 7y, and 10y. The table also reflects some of the interpolated CDS spreads. For the interval (0,6m], it could be presumed either a flat TSCDS or the same slope as in the interval [6m,1y]. For this and further examples, the latter option is adopted.[7] It is also assumed a constant risk-free rate (2% in this case) and a recovery rate of 40%. The term structure of CRDF is instantaneously obtained with the sole help of a spreadsheet (i.e., no root-search algorithm is required). The outcome for the

[5] If so required, the term structure of risk-neutral survival probabilities could be also obtained as a "sub-product" of the bootstrapping process: $S(T) = C(T)/Z(T)$. That said, such additional result is not necessary for any of the applications considered in the following section.

[6] The Excel file containing this example is available at www.santiagoforte.com.

[7] Caution should be taken to prevent that this interpolation leads to negative CDS spreads in the interval (0,6m]. However, this does not happen in any of the cases explored.

selected maturities is incorporated into Table 1, while Figure 4 reproduces the full-term structure of CRDF. The following section describes only a few of the many possible applications of this simple pricing method.

<Table 1 about here>

<Figure 4 about here>

5. Applications

The most evident application of the term structure of CRDF is the mark-to-market of any position in a CDS contract using Equation (6). By extension, this also implies a simple approach to estimating CDS returns (Berndt and Obreja, 2010; Augustin, Saleh and Xu, 2020). Other possible applications are as follows:

5.1. Pricing of Risky Bonds

Consider a risky bond with coupon b, nominal p, and maturity T. Let us also denote T_m the maturity of the mth coupon payment, where $m = 1, \dots, M$, and $T_M = T$. The present value of this bond will be:

$$d(T) = b \sum_{m=1}^{M} C(T_m) + pC(T) + \theta p B(T). \tag{10}$$

The first term in the right-hand side of previous equation reflects the present value of the stream of coupon payments. The second term accounts for the payment of the nominal at maturity in the case of no default. Finally, the last term incorporates the present value of the fractional recovery of the nominal value in the case of default.

5.2. Pricing of Forward CDS

Now, consider a forward CDS contract signed at current time 0 for credit protection between T_j and T_k, with $0 \leq T_j < T_k$. More precisely, the initiation date is T_j conditional on $\tau^d > T_j$, so the first effective date with accrual of premium payments and delivery of the bond in exchange of the bond's face value in the case of default is $T_j + \Delta$. The daily structure of this contract is, in fact, the same as already described in Figures 2 and 3 for a spot contract. The sole difference is that the starting date is now T_j rather than 0, and the ending date is T_k. To derive the present value of the premium leg of the forward contract based on the CRDF, let us define (for any T^* and T, with $0 \leq T^* < T$):

- $A(T^*, T)$: The present value of the same asset class A paying a constant annuity of \$$\Delta$ every Δ years, but this time between T^* and T with the following conditions: (i) the first payment is at $T^* + \Delta$, conditional on $\tau^d > T^*$ (otherwise, the asset is liquidated at τ^d), and; (ii) provided that $\tau^d > T^*$, the last payment is at L_d^T (included).

From the definition of $A(T)$ and $A(T^*, T)$, it holds that

$$A(T^*, T) = A(T) - A(T^*), \tag{11}$$

and if we use $fcds$ to denote the spread of the forward CDS contract described above, the present value of the premium leg is:

$$X(T_j, T_k) = fcds A(T_j, T_k). \tag{12}$$

We can also derive the present value of the protection leg based on the CRDF. Let us define:

- $B(T^*, T)$: The present value of the same asset class B paying \$1 at τ^d, provided this time that $T^* < \tau^d \leq T$.

From the definition of $B(T)$ and $B(T^*, T)$, it must hold that

$$B(T^*, T) = B(T) - B(T^*), \tag{13}$$

and the present value of the protection leg is:

$$Y(T_j, T_k) = (1 - \theta)B(T_j, T_k). \tag{14}$$

The value of a long position in the forward CDS contract is in this way:

$$FV(T_j, T_k) = (1 - \theta)B(T_j, T_k) - fcdsA(T_j, T_k). \tag{15}$$

By imposing $FV(T_j, T_k) = 0$, we finally obtain the break-even forward CDS spread:

$$fcds(T_j, T_k) = \frac{(1 - \theta)B(T_j, T_k)}{A(T_j, T_k)}. \tag{16}$$

It is worth noting that $fcds(0, T) = cds(T)$.

5.3. Time Decomposition of CDS Spreads

The time decomposition of a CDS spread refers to the problem of determining the percentage of the spread that can be reasonably attributed to the protection of specific time intervals within the contract's maturity. Despite its intrinsic interest, this question has not received much attention in the academic literature. As shown below, the time decomposition of CDS spreads exhibits similarities, but also remarkable differences compared to the traditional decomposition of spot (risk-free interest) rates into forward rates.

In a similar vein to a spot rate decomposition, the time decomposition of CDS spreads follows from the possibility of representing a long (short) position in a CDS contract as a portfolio of long (short) positions in N consecutive forward CDS contracts. If we define $T_0 = 0$ and $T_N = T$, then

$$X(T) = \sum_{i=1}^{N} X(T_{i-1}, T_i), \tag{17}$$

that is, the present value of the cost of credit protection up to time T must be equal to the present value of the cost of credit protection for an arbitrary number of consecutive (not necessarily identical) time intervals between time 0 and time T. Consequently,

$$cds(T) = \sum_{i=1}^{N} w(T_{i-1}, T_i; T) fcds(T_{i-1}, T_i), \tag{18}$$

where

$$w(T_{i-1}, T_i; T) = \frac{A(T_{i-1}, T_i)}{A(T)} \in [0,1]; \tag{19}$$

with

$$\sum_{i=1}^{N} w(T_{i-1}, T_i; T) = 1. \tag{20}$$

From Equations (18–20), it is concluded that we can divide the maturity T of a CDS contract into an arbitrary number of intermediate time intervals and express the associated CDS spread as a weighted average of the forward CDS spreads corresponding to those timeslots. The weight of a particular forward spread $fcds(T_{i-1}, T_i)$ on the spot spread $cds(T)$ is given by the weight of $A(T_{i-1}, T_i)$ into $A(T)$. Among the factors that will influence this ratio (relative time length, time value of money), it is worth highlighting the risk of default up to the initiation date T_{i-1}. Other things equal, the higher this risk, the lower the present value of any stream of payments in the time interval $(T_{i-1}, T_i]$ conditional on no-previous default, and, therefore, the lower the influence of the corresponding forward spread on the spot spread. This reflects one main difference concerning the time decomposition of spot rates: contrary to the forward rates embedded in a spot rate, the forward CDS spreads contained in a CDS spread may never be paid, and this will be properly reflected in their weights. In summary, the risk of default will enter both the forward spreads and their weights.

As a corollary to previous results, the level and steepness of the TSCDS will be closely related. To see this more clearly, let us consider the following simple decomposition:

$$cds(T) = \left[1 - \frac{A(T^*, T)}{A(T)}\right] cds(T^*) + \frac{A(T^*, T)}{A(T)} fcds(T^*, T); \tag{21}$$

and rearranging terms,

$$[cds(T) - cds(T^*)] = \frac{A(T^*, T)}{A(T)} [fcds(T^*, T) - cds(T^*)]. \tag{22}$$

Equation (22) provides an intuitive interpretation of the possible forms of the TSCDS (increasing, decreasing, or hump-shaped) and its steepness. Concerning the possible forms, a forward spread $fcds(T^*, T)$ higher (lower) than the spot spread $cds(T^*)$ will imply, as expected, a positive (negative) slope in the interval $[T^*, T]$. However, the steepness will not only depend on the absolute difference between $fcds(T^*, T)$ and $cds(T^*)$, but also on the ratio $A(T^*, T)/A(T)$. Following previous arguments, the higher the risk of default up to T^*, the lower this ratio and, assuming other things equal, the flatter the TSCDS in the interval $[T^*, T]$. Thus, in effect, when it comes to analyzing a TSCDS, level and steepness cannot be dissociated. These results are consistent with predictions based on hazard rates and previous empirical evidence (Lando and Mortensen, 2005).[8]

A final implication from previous results is the possibility to interpret $w(T_{i-1}, T_i; T) fcds(T_{i-1}, T_i)$ as the total contribution of the time interval $(T_{i-1}, T_i]$ to the CDS spread with maturity T. Accordingly, the relative contribution will be

$$Q(T_{i-1}, T_i; T) = \frac{w(T_{i-1}, T_i; T) fcds(T_{i-1}, T_i)}{cds(T)} = \frac{B(T_{i-1}, T_i)}{B(T)} \in [0,1], \tag{23}$$

with

$$\sum_{i=1}^{N} Q(T_{i-1}, T_i; T) = 1. \tag{24}$$

Equation (23) indicates that the relative contribution of one particular time interval $(T_{i-1}, T_i]$ to the spread of a CDS contract with maturity T is given by the ratio between the following: the present value of \$1 paid at default if this happens during that particular time interval, and the present value of \$1 paid at default if this happens at any time during the life of the contract. The proximity of these two values would imply that the risk of default is concentrated in that specific time interval, thereby resulting in a significant

[8] Practitioners usually think of forward CDS spreads as a function of spot spreads. Likewise, the term structure of forward CDS spreads is typically analyzed based on the TSCDS. From a practitioner's point of view, this makes full sense. Trading normally concentrates on liquid spot contracts, so, if needed, a forward contract can be constructed synthetically from such spot contracts. However, from a strict economic interpretation, it makes more sense to think of spot spreads as a product of forward spreads.

contribution to the spread of the CDS contract. Evidently, the opposite result will be achieved if there is a significant difference between the two aforementioned values.

5.4. Possible Extension to Portfolio Management

Previous results concentrate on the pricing of single-name credit risky securities at current (non-defaulting) time 0. However, they can be easily extended to portfolio management. As it has been shown, the combination of the TSCDS and the term structure of risk-free interest rates (TSIR) provides a direct estimate of the term structure of CRDF. Moreover, the prices of risky bonds, CDS contracts, and forward CDS contracts are simple functions of those CRDF. Consequently, the method provides a direct mapping between *observable* market risk factors (TSCDS and TSIR) and the prices of the most common single-name credit risky securities. The final implication is the possibility to translate the predicted distribution function for those market risk factors into a distribution function for the value of different credit risky portfolios using Monte Carlo simulations, which also represents and easy way path for the integration of market and credit risk.[9]

6. Case Study: The Eurozone Crisis

The Eurozone crisis provides an interesting framework to illustrate the pricing method presented in this study and some of its applications. It combines, in a short time, issuers with relatively low and extremely high CDS spread levels. We may also expect the liquidity of a CDS contract to be higher for France or Ireland than for an average corporation with a similar credit risk level. For the analyses that follow, CDS spreads with maturities ranging from 6-months to 5-years and a CR/CR14 restructuring clause are collected from Markit. It is assumed a constant risk-free rate equal to the 5-years swap

[9] Clearly, this extension to portfolio management should incorporate the probability of a default event at the future pricing date. More precisely, the empirical evidence on the connection between historical/current CDS levels and the probability of a future default event should be accounted for.

rate and a recovery rate of 40%. Regarding the estimated forward CDS spreads, they will always refer to contracts with a total length of 1-year initiated either at time 0 (identical to a 1-year spot spread), 1, 2, 3, or 4.

6.1. Examples of the Term Structure of CDS Spreads

Figure 5 analyzes the case of France (March 15, 2011) and contains six panels. Panel 5A plots the TSCDS and reflects the effectively observed CDS spreads and the interpolated spreads. The resulting term structures of $A(T)$, $B(T)$, and $C(T)$ values are presented in Panels 5B, 5C, and 5E, respectively. Taken together, the information provided in these panels is representative of an investment-grade issuer. To begin, the overall level of the TSCDS is low and has a positive slope, which reflects the higher uncertainty associated with future time periods. Additionally, the smooth decline in $C(T)$ as the maturity increases seems to be more a reflection of the time value of money than of a significant growth in the risk of default. Consistent with this perception, $B(T)$ remains at low levels whereas $A(T)$ grows steadily. Panel 5E again depicts the TSCDS, although this time in combination with the estimated term structure of forward CDS spreads (TSFCDS) and, for comparison purposes, the term structure of the simple mean of forward CDS spreads (TSMFCDS). The panel also contains the actual weight of each forward CDS spread in the different spot spreads. In this particular example, such weights are always close to $1/T$; that is, all relevant forward CDS spreads have approximately the same influence on a given CDS spread. Consequently, the TSCDS is very similar to the TSMFCDS. Panel 5F provides the final contribution of each year of protection on each CDS spread. If we focus on the time decomposition of the 5-year CDS spread, the exact contributions of years 1, 2, 3, 4, and 5 are 8%, 13%, 19%, 28%, and 31%, respectively. As already reflected in Panel 5E, such differences are explained by the TSFCDS alone.

The actual weight of each forward spread on the 5-year spot spread is roughly the same, and, in fact, slightly decreasing.

<Figure 5 about here>

Figure 6 reproduces Figure 5 in the case of Greece (September 13, 2011) and reveals a completely different situation. In Panel 6A, the risk of imminent default is already reflected in the rather extreme short-run CDS spreads.[10] Consistent with this situation, the $C(T)$ value (Panel 6D) declines rapidly to reach just 49 cents of a euro for the maturity of 6-months and 10 cents for the maturity of 5-years. The $B(T)$ value (Panel 6C) moves in the opposite direction: €0.50 for 6-months, and €0.88 for 5-years. Moreover, for those same maturities the $A(T)$ value (Panel 6B) reaches only 0.35 and 1.20 euros, respectively. The results in Panel 6E reflect the predicted connection between level and steepness in the TSCDS (section 5.3). Namely, because of the low present value of future payments conditional on no-previous default, the corresponding forward spreads have a small weight on the spot spreads, and this, in turn, translates into a TSCDS that is significantly flatter than the TSMFCDS. For instance, while the weight of the first forward CDS spread in the 5-year spot CDS spread was 22% in the example of France, this same weight jumps to 46% in the case of Greece. This effectively explains why the significant drop in successive forward CDS spreads does not translate into a proportional reduction in spot spreads. Finally, as reflected in Panel 6F, the combination of a high first forward CDS spread (in fact, the highest one) and a high weight for this spread makes the

[10] The initial proposal of a bond exchange with a nominal discount of 50% on notional Greek debt was made in the Euro Summit held on October 26, 2011, and was formally announced on February 21, 2012 (see Zettelmeyer, Trebeschand and Gulati (2013) for details). On February 28, 2012, the International Swaps and Derivatives Association (ISDA) accepted a question related to a potential Hellenic Republic credit event. The occurrence of a credit event was initially denied by the ISDA on March 1, 2012, but was finally accepted on March 9, 2012, after a second question was formulated.

protection of year one account for 76% of the 5-year CDS spread. This is in sharp contrast to the corresponding value of 8% in the example of France.

<Figure 6 about here>

Figure 7 repeats the same analysis for the case of Ireland (October 25, 2011). It provides an example of an intermediate, hump-shaped TSCDS that, for the rest, is consistent with previous results.

<Figure 7 about here>

6.2. The European Central Bank Intervention

"The ECB is ready to do whatever it takes to preserve the Euro, and believe me; it will be enough."

Mario Dragui, President of the European Central Bank. July 26, 2012.

The risk of a collapse in the Eurozone forced the European Central Bank (ECB) to change its policy. It is well known that Dragui's remark on July 26, 2012, and the decisions that followed had a significant impact on the credit spreads within the Eurozone. We can now analyze this impact in more detail. For ease of exposition and to save space, the analysis concentrates on the case of Spain. The results for other countries (e.g., Italy, Ireland, and Portugal) exhibit a similar pattern.

Table 2 contains the main descriptive statistics for the 6-month to 5-year CDS spreads on a weekly basis from January 2010 to December 2019.[11] Figure 8 shows the analysis' results and is composed of four panels. Panels 8A and 8B depict the evolution of spot and forward CDS spreads, and they both reflect the significant impact of Dragui's

[11] Zero and negative 5-year swap rates are sometimes observed starting March 2016. To avoid potential problems associated with non-positive risk-free interest rates, a minimum value of 0.01% is imposed.

statement on the cost of credit protection. Panels 8C and 8D focus on the composition of the 5-year CDS spreads. Regarding the weight of each forward spread (Panel 8C), the first forward CDS spread's weight reaches its pick immediately before the announcement, while the weight of the last forward spread reaches its bottom. After Dragui's remark, the weight of all forward spreads starts to converge until finally reflecting the situation of an investment-grade issuer (see the example of France). The evolution of each year's contribution to the 5-year CDS spread (Panel 8D) confirms that the statement and the posterior policy change in the ECB did not only have a significant impact on the level, but also on the composition of the CDS spreads.

<Table 2 about here>

<Figure 8 about here>

7. Conclusions

This study introduces a simple no-arbitrage approach to pricing single-name credit risky securities that circumvents the estimation of risk-neutral default probabilities or hazard rates. Similar to the traditional estimation of implied discount factors in risk-free bond prices with different maturities, the method provides a direct estimate of credit risk discount factors from the term structure of credit default swap spreads. The proposal could thereby be seen as a "back to basics" exercise in credit risk pricing, based in fact on a financial innovation: the increasing liquidity of the full-term structure of credit default swap spreads. The possibility of adapting the method to other scenarios represents an interesting topic for further research.

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Tables and Figures

Table 1. Numerical example of the bootstrapping of credit risk discount factors.

Obs. Mat.	T	$cds(T)$	$A(T)$	$B(T)$	$C(T)$
	0	**-**	**0.00000**	**0.00000**	**1.00000**
	1/365	52.00	0.00274	0.00002	0.99992
	2/365	52.13	0.00548	0.00005	0.99984
	…	…	…	…	…
	182/365	74.87	0.49477	0.00617	0.98393
6m	**183/365**	**75.00**	0.49746	0.00622	0.98383
	184/365	75.13	0.50016	0.00626	0.98373
	…	…	…	…	…
	364/365	97.87	0.98065	0.01600	0.96439
1y	**1**	**98.00**	0.98329	0.01606	0.96427
	…	…	…	…	…
2y	**2**	**135.00**	1.92535	0.04332	0.91817
	…	…	…	…	…
3y	**3**	**160.00**	2.81911	0.07518	0.86844
	…	…	…	…	…
4y	**4**	**179.00**	3.66234	0.10926	0.81749
	…	…	…	…	…
5y	**5**	**192.00**	4.45534	0.14257	0.76832
	…	…	…	…	…
7y	**7**	**205.00**	5.90342	0.20170	0.68023
	…	…	…	…	…
10y	**10**	**212.00**	7.77503	0.27472	0.56978

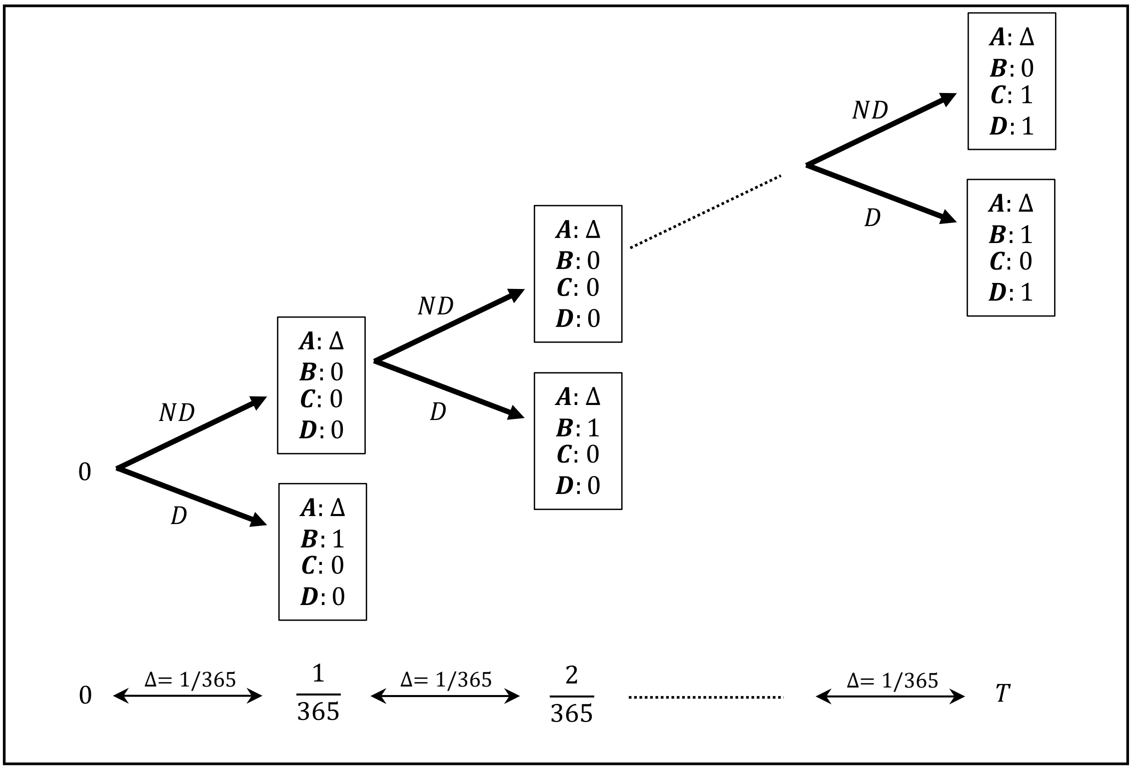
This table provides a numerical example of estimating the term structure of $A(T)$, $B(T)$, and $C(T)$ based on the TSCDS and no-arbitrage conditions. Daily CDS spreads are obtained by linear interpolating CDS spreads with observed maturities (Obs. Mat.). Typically observed maturities and the initial values are indicated in bold format. It is assumed a constant risk-free rate of 2%, and a recovery rate of 40%.

Table 2. Main descriptive statistics for the CDS spreads of Spain.

Statistic	cds(0.5)	cds(1)	cds(2)	cds(3)	cds(4)	cds(5)
Mean	70.88	80.51	98.93	112.73	122.11	130.51
Median	23.30	31.13	47.49	60.21	70.79	80.76
Min	3.64	5.25	9.58	13.72	17.79	24.02
Max	375.53	409.00	486.14	503.25	504.31	504.15
SD	88.78	93.07	101.95	105.10	104.26	103.08

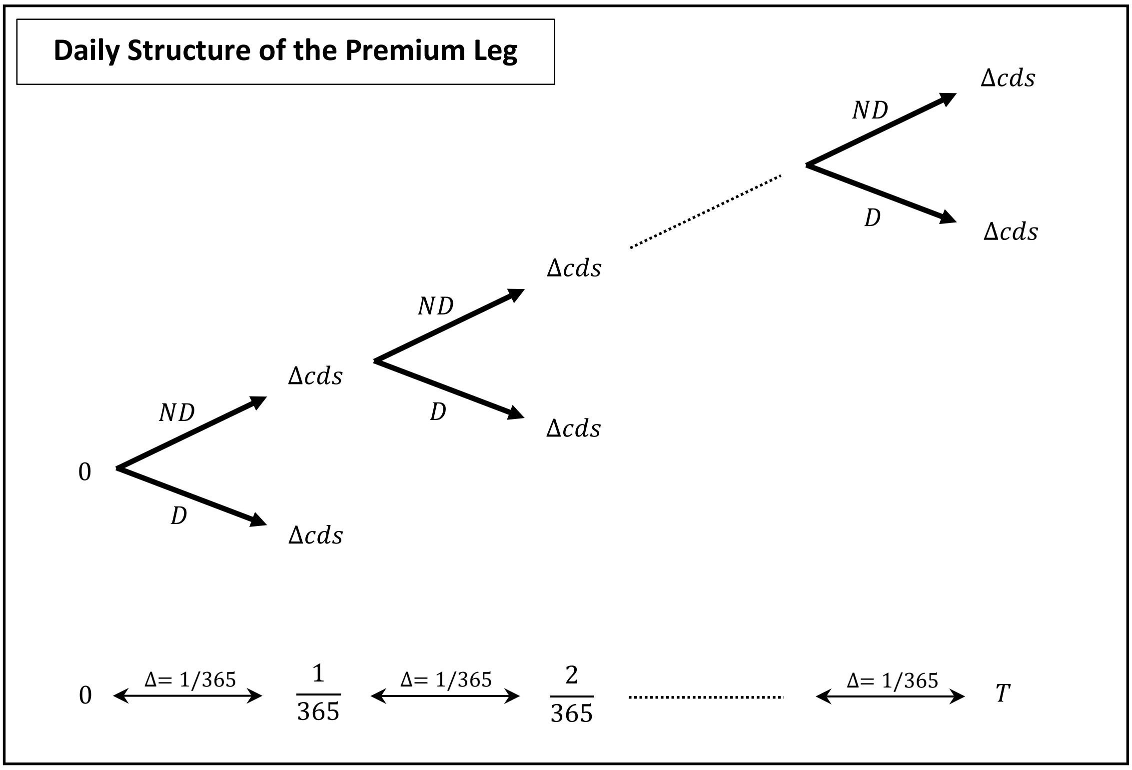
This table provides the main descriptive statistics for the CDS spreads of Spain. Data are collected weekly for the inclusive period from January 2010 to December 2019.

Figure 1. Structure of payments for assets A, B, C, and D with maturity T.



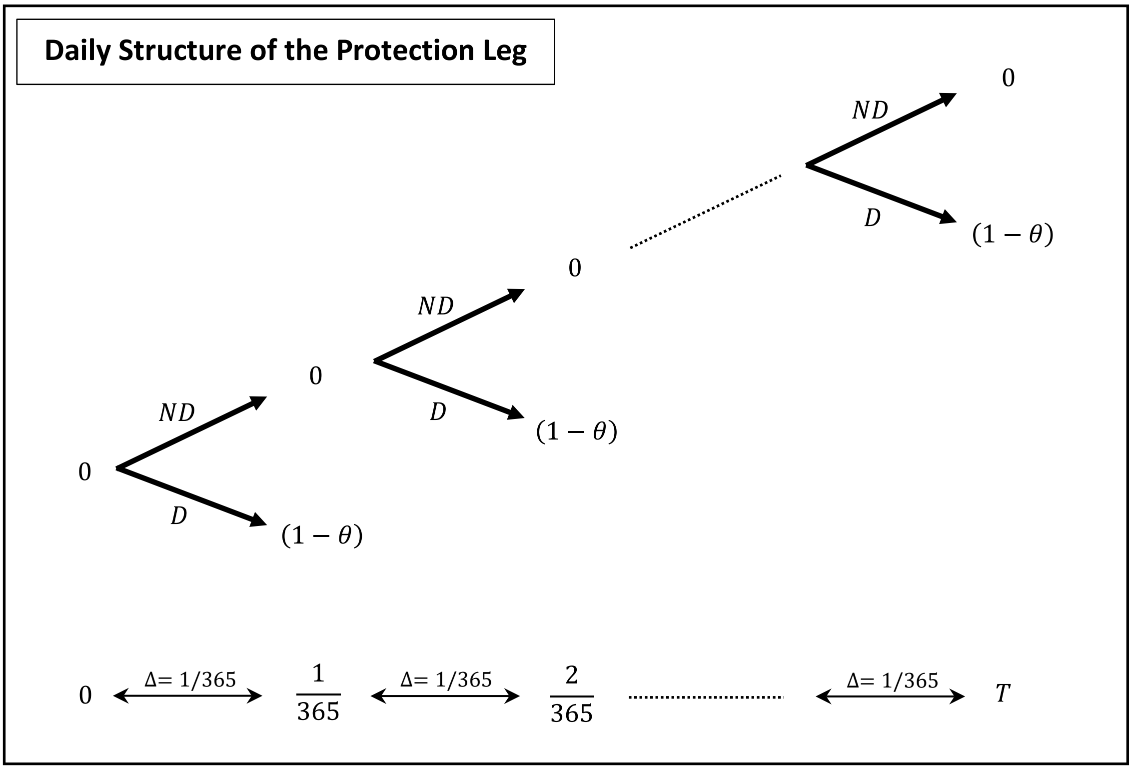
This figure describes the structure of payments for assets A, B, C, and D with maturity $T > 0$. The possible outcomes each day are no default (ND) or default (D).

Figure 2. Daily structure of the premium leg in a CDS contract with maturity T.



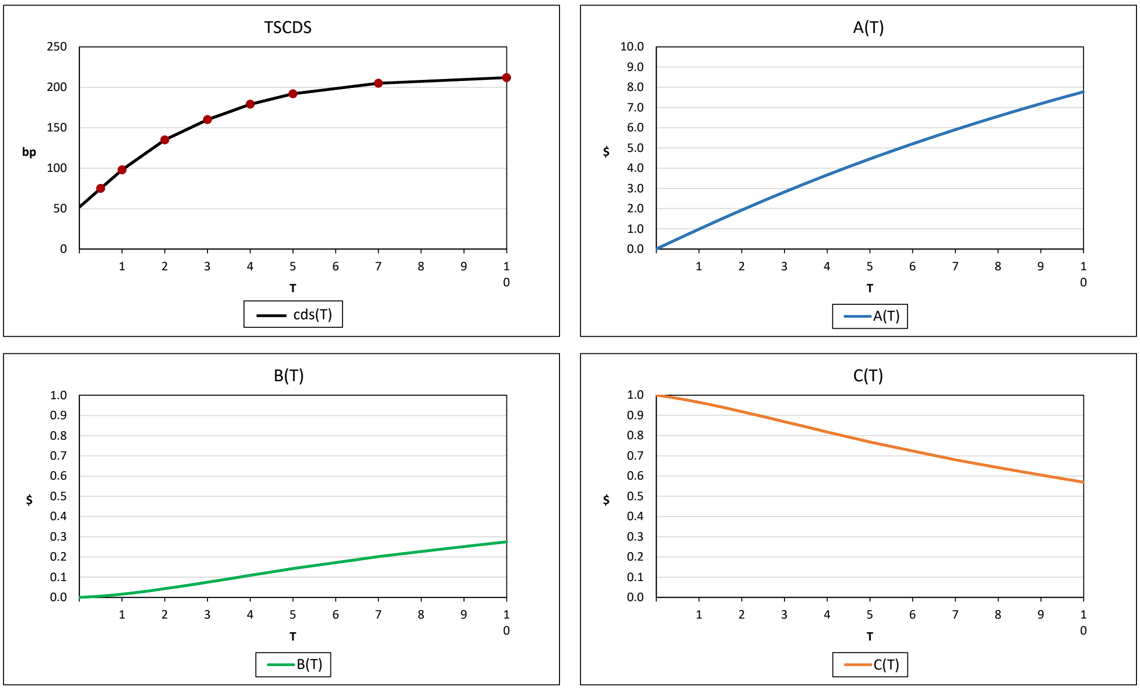
This figure describes the daily structure of the premium leg in a CDS contract with maturity $T > 0$. The possible outcomes each day are no default (ND) or default (D).

Figure 3. Daily structure of the protection leg in a CDS contract with maturity *T*.



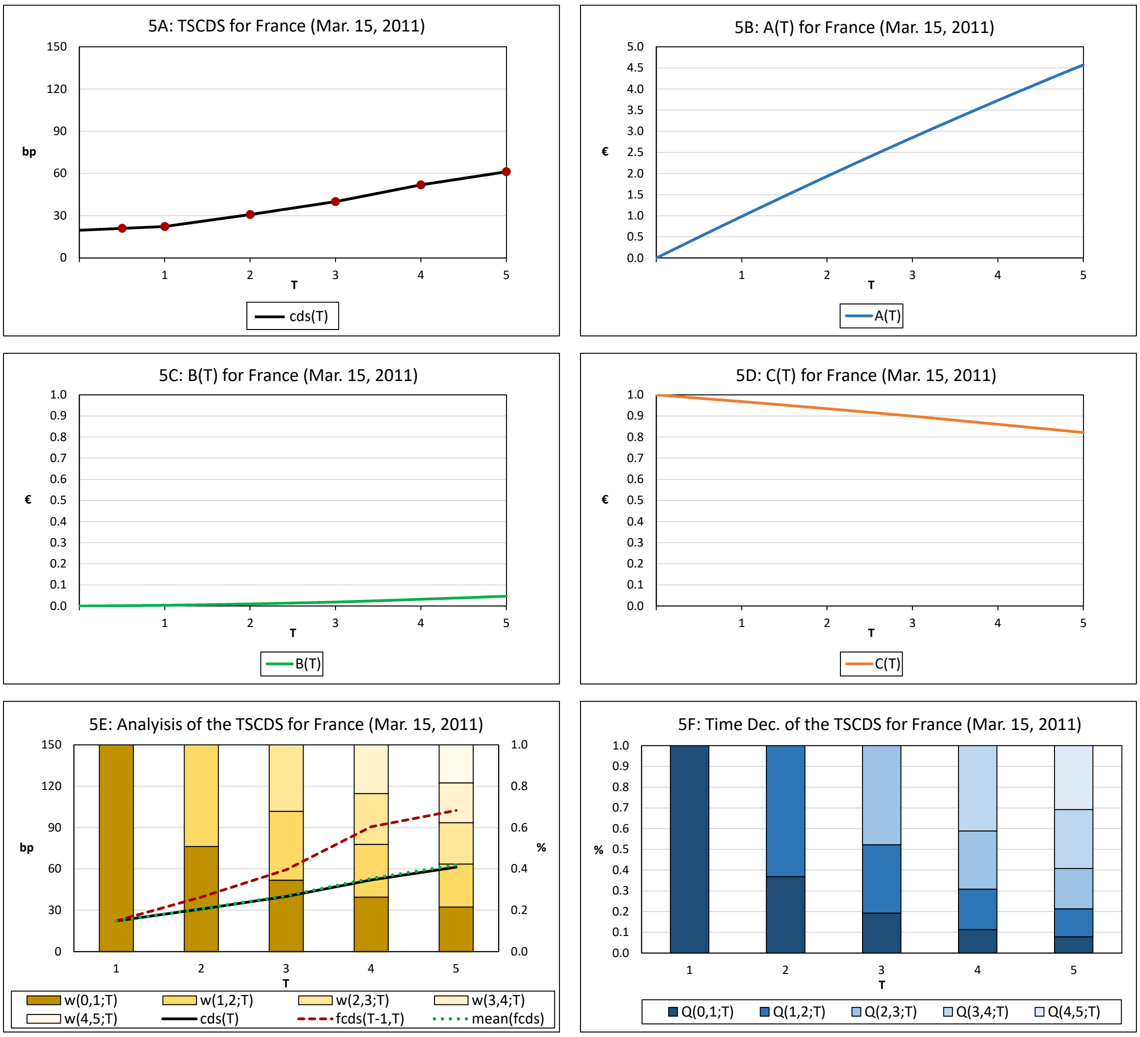
This figure describes the daily structure of the protection leg in a CDS contract with maturity $T > 0$. The possible outcomes each day are no default (ND) or default (D).

Figure 4. Numerical example of the bootstrapping of credit risk discount factors.



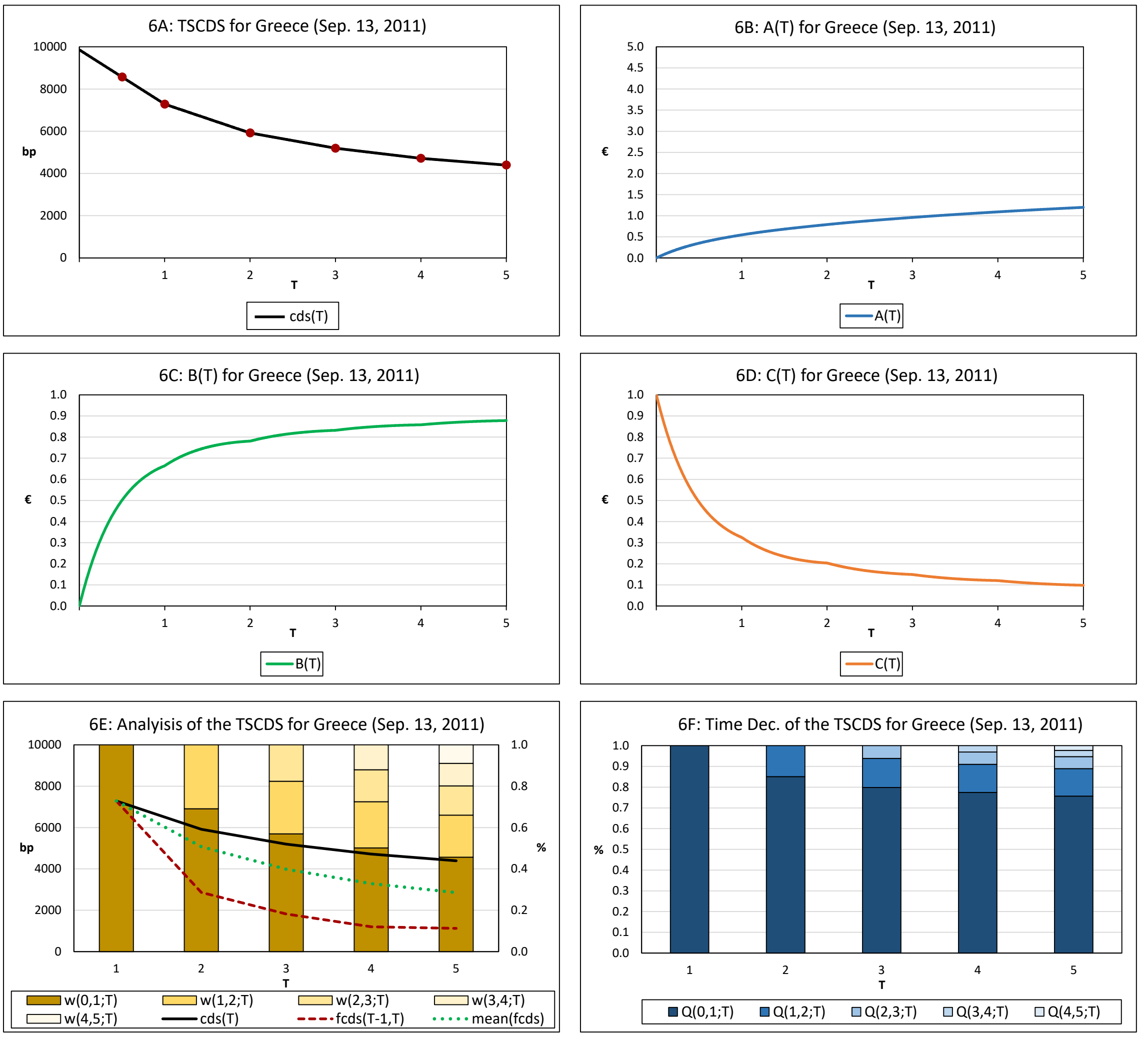
This figure plots the results of the numerical example, where the term structure of $A(T)$, $B(T)$, and $C(T)$ is estimated based on the TSCDS and no-arbitrage conditions. The red point indicates that the CDS spread corresponds to a typically observed maturity: 6m, 1y, 2y, 3y, 4y, 5y, 7y, or 10y.

Figure 5. Analysis and time decomposition of the TSCDS for France - March 15, 2011.



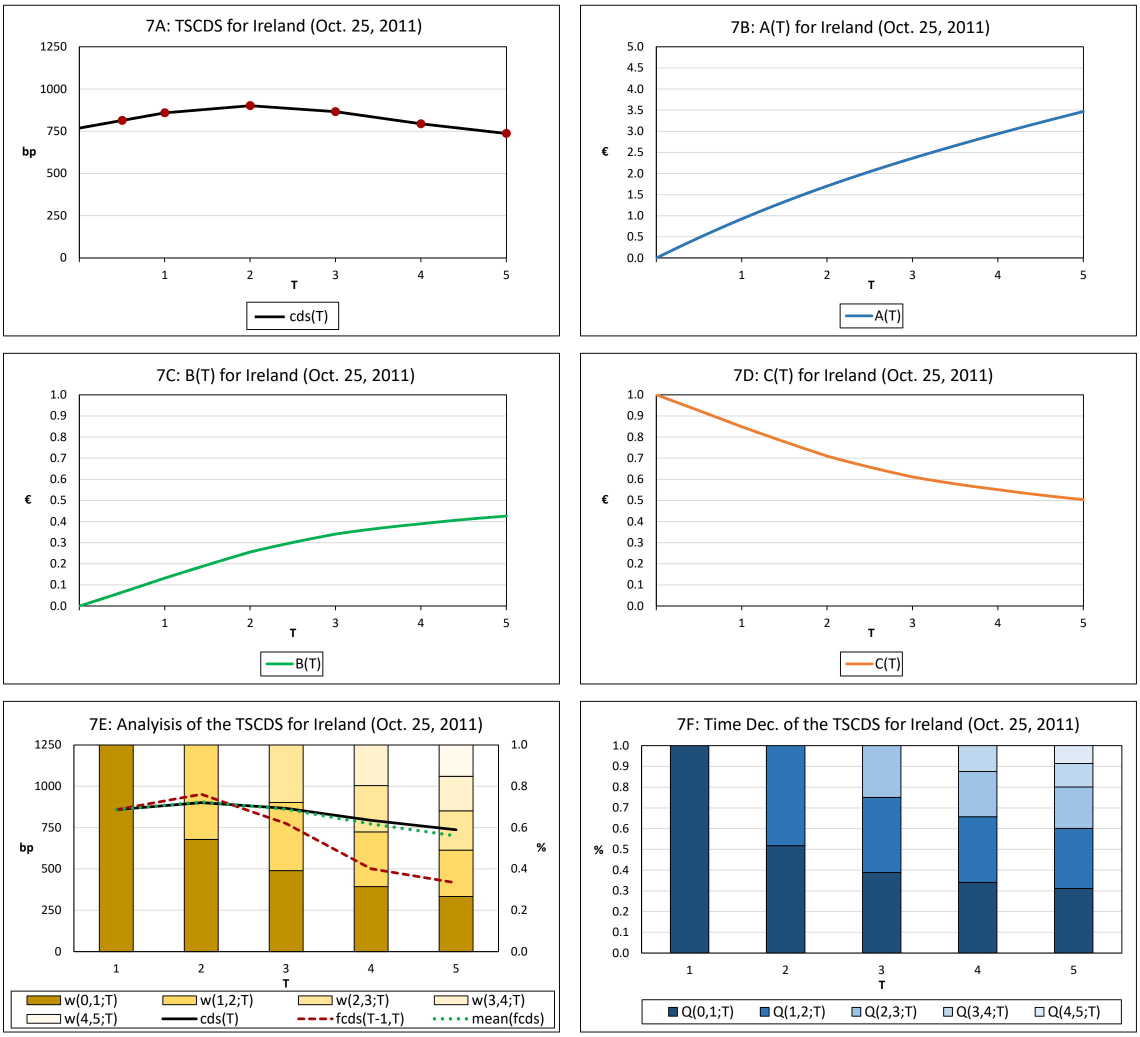
This figure shows the results of the analysis and time decomposition of the TSCDS for France on March 15, 2011. Panel 5A plots the TSCDS (the red point indicates an observed CDS spread). Panels 5B, 5C, and 5D depict the term structures of $A(T)$, $B(T)$, and $C(T)$, respectively. Panel 5E plots the TSCDS, TSFCDS, and TSMFCDS. The actual weight of each forward CDS spread on each spot CDS spread is also provided. Panel 5F shows the final decomposition of each CDS spread.

Figure 6. Analysis and time decomposition of the TSCDS for Greece - September 13, 2011.



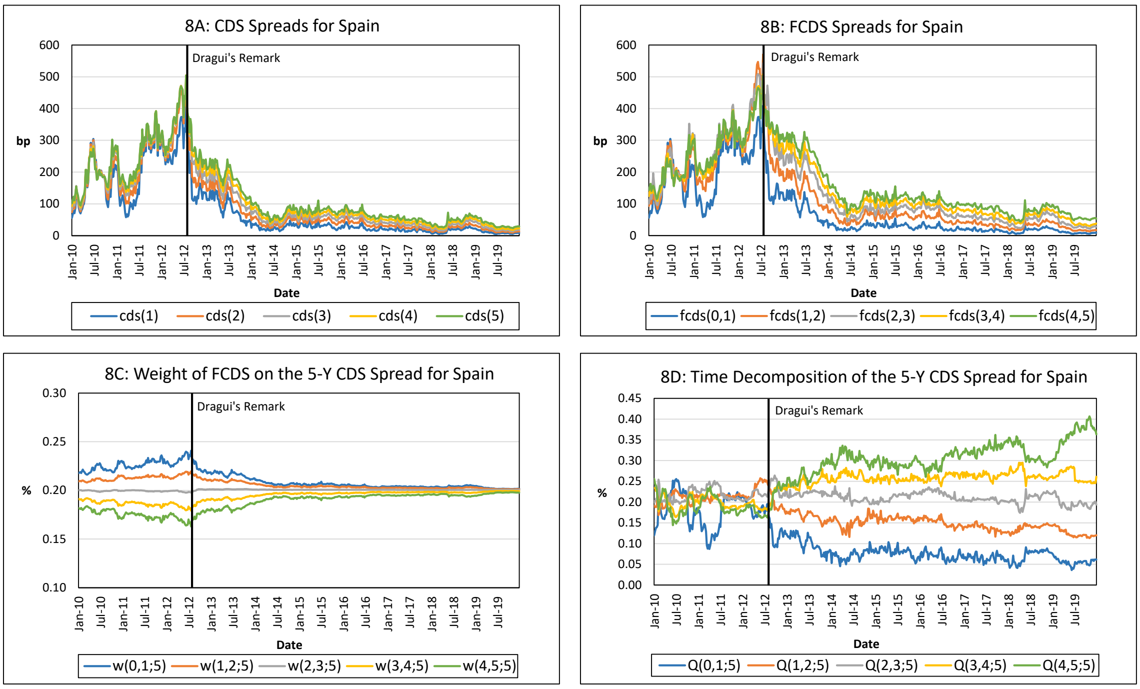
This figure shows the results of the analysis and time decomposition of the TSCDS for Greece on September 13, 2011. Panel 6A plots the TSCDS (the red point indicates an observed CDS spread). Panels 6B, 6C, and 6D depict the term structures of $A(T)$, $B(T)$, and $C(T)$, respectively. Panel 6E plots the TSCDS, TSFCDS, and TSMFCDS. The actual weight of each forward CDS spread on each spot CDS spread is also provided. Panel 6F shows the final decomposition of each CDS spread.

Figure 7. Analysis and time decomposition of the TSCDS for Ireland - October 25, 2011.



This figure shows the results of the analysis and time decomposition of the TSCDS for Ireland on October 25, 2011. Panel 7A plots the TSCDS (the red point indicates an observed CDS spread). Panels 7B, 7C, and 7D depict the term structures of $A(T)$, $B(T)$, and $C(T)$, respectively. Panel 7E plots the TSCDS, TSFCDS, and TSMFCDS. The actual weight of each forward CDS spread on each spot CDS spread is also provided. Panel 7F shows the final decomposition of each CDS spread.

Figure 8. Analysis and time decomposition of the TSCDS for Spain. January 2010 – December 2019.



This figure shows the results of the analysis and time decomposition of the TSCDS for Spain within the inclusive period from January 2010 to December 2019. Panel 8A plots the time series of one to 5-year CDS spreads. Panel 8B depicts the time series of forward CDS spreads. Panel 8C describes the evolution of the weight of each forward CDS spread on the 5-year CDS spread. Panel 8D plots the time series of the final decomposition of the 5-year CDS spread.